

Programming #2: Part I (4190.410)

Due: September 21, 2015

Given a cubic Bézier curve $C(t) = \sum_{l=0}^3 \mathbf{b}_l B_l^3(t)$, $0 \leq t \leq 1$, with four control points $\mathbf{b}_0, \mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3$, we have the following differential properties:

$$\begin{aligned} C(t) = (x(t), y(t)) &= (1-t)^3 \mathbf{b}_0 + 3(1-t)^2 t \mathbf{b}_1 + 3(1-t)t^2 \mathbf{b}_2 + t^3 \mathbf{b}_3, \\ C'(t)/3 = (x'(t), y'(t))/3 &= (1-t)^2 (\mathbf{b}_1 - \mathbf{b}_0) + 2(1-t)t (\mathbf{b}_2 - \mathbf{b}_1) + t^2 (\mathbf{b}_3 - \mathbf{b}_2) \\ &= (\mathbf{b}_1 - \mathbf{b}_0) + t (2\mathbf{b}_2 - 4\mathbf{b}_1 + 2\mathbf{b}_0) + t^2 (\mathbf{b}_3 - 3\mathbf{b}_2 + 3\mathbf{b}_1 - \mathbf{b}_0), \\ C''(t)/6 = (x''(t), y''(t))/6 &= (1-t) (\mathbf{b}_2 - 2\mathbf{b}_1 + \mathbf{b}_0) + t (\mathbf{b}_3 - 2\mathbf{b}_2 + \mathbf{b}_1) \\ &= (\mathbf{b}_2 - 2\mathbf{b}_1 + \mathbf{b}_0) + t (\mathbf{b}_3 - 3\mathbf{b}_2 + 3\mathbf{b}_1 - \mathbf{b}_0). \end{aligned}$$

Part I: Compute all x and y -extreme points as well as all inflection points of $C(t)$, $0 \leq t \leq 1$, by solving the following three quadratic equations:

$$x'(t)/3 = 0, \quad y'(t)/3 = 0, \quad (x''(t)y'(t) - x'(t)y''(t))/18 = 0.$$