Quiz #2 (CSE 400.001)

Tuesday, March 26, 2002

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1. (10 points) Solve the following differential equation

$$y'' - 2y' + 5y = e^x \cos 2x.$$

$\lambda^2 - 2\lambda + 5 = (\lambda - 1)^2 + 2^2 = 0$, $\lambda = 1 \pm 2i$

$y_h = A e^x \cos 2x + B e^x \sin 2x$ (+1)

$y_p = x e^x (K \cos 2x + M \sin 2x)$ (+2)

$y_p' = (e^x + x e^x)(K \cos 2x + M \sin 2x) + x e^x (-2K \sin 2x + 2M \cos 2x)$ (+1)

$y_p'' = (2e^x + x e^x)(K \cos 2x + M \sin 2x) + (2e^x + 2x e^x)(-2K \sin 2x + 2M \cos 2x) + x e^x (-4K \cos 2x - 4M \sin 2x)$ (+1)

$y_p'' - 2y_p' + 5y_p = e^x (-2K \sin 2x + 2M \cos 2x) = e^x \cos 2x$

$\therefore K = 0$, $M = \frac{1}{4}$ (+4)

$\therefore y = A e^x \cos 2x + B e^x \sin 2x + \frac{1}{4} x e^x \sin 2x$ (+1)

2. (10 points) Solve the following initial value problem.

$$x^2y'' - 3xy' + 3y = 24x^4e^x, \quad y(1) = 2, \ y'(1) = 24e.$$

$$m(m-1) - 3m + 3 = m^2 4m + 3 = (m-1)(m-3) = 0 \quad +1$$

$$\therefore y_1 = x, \quad y_2 = x^3$$

$$W = \begin{vmatrix} x & x^3 \\ 1 & 3x^2 \end{vmatrix} = 2x^3 \quad +1$$

$$y'' - \frac{3}{x}y' + \frac{3}{x^2}y = 24x^2e^x \quad +2$$

$$y_p = -y_1\int \frac{y_2}{W} r(x)\,dx + y_2 \int \frac{y_1}{W} r(x)\,dx \quad +2$$

$$= -x\int \frac{x^3}{2x^3} \cdot 12x^2e^x dx + x_3 \int \frac{x}{2x^3} \cdot 12x^2 e^x dx$$

$$= -12x \int x^2 e^x dx + 12x^3 \int e^x dx$$

$$= (-12x)\left[\cancel{x^2e^x} - 2\int xe^x dx\right] + \cancel{12x^3e^x}$$

$$= (24x)\left[xe^x - \int e^x dx\right]$$

$$= 24x^2e^x - 24xe^x \quad +2$$

$$\begin{cases} y = c_1x + c_2x^3 + 24x^2e^x - 24xe^x \\ y' = c_1 + 3c_2x^2 + 24x^2e^x + 24xe^x - 24e^x \end{cases}$$

$$\begin{cases} c_1 + c_2 = 2 \\ c_1 + 3c_2 + 24e = 24e \end{cases} \Rightarrow \begin{cases} c_1 + c_2 = 2 \\ c_1 + 3c_2 = 0 \end{cases} \quad +2$$

$$\therefore c_1 = 3, \ c_2 = -1$$

$$\therefore y = 3x - x^3 + 24x^2e^x - 24xe^x$$

3. (10 points) Solve the following initial value problem

$$y''' + y'' = 10e^x \cos x, \quad y(0) = 0,\ y'(0) = 0,\ y''(0) = 0.$$

$\lambda^3 + \lambda^2 = \lambda^2(\lambda+1) = 0, \quad \lambda = 0, -1,$ (+1)

$y_h = c_1 + c_2 x + c_3 e^{-x}$ (+2)

$y_p = e^x (K\cos x + M\sin x)$ (+1)

$$\left.\begin{aligned} y_p' &= e^x(K\cos x + M\sin x) + e^x(-K\sin x + M\cos x) \\ y_p'' &= e^x\cancel{(K\cos x + M\sin x)} + 2e^x(-K\sin x + M\cos x) \\ &\quad + e^x\cancel{(-K\cos x - M\sin x)} \\ y_p''' &= 2e^x(-K\sin x + M\cos x) + 2e^x(-K\cos x - M\sin x) \end{aligned}\right] (+2)$$

$$\left.\begin{aligned} y_p''' + y_p'' &= 4e^x(-K\sin x + M\cos x) + 2e^x(-K\cos x - M\sin x) \\ &= 10e^x\cos x \\ &\begin{cases} -4K - 2M = 0 \\ 4M - 2K = 10 \end{cases} \qquad \therefore K = -1,\ M = 2 \end{aligned}\right] (+2)$$

$$\begin{cases} y = c_1 + c_2 x + c_3 e^{-x} - e^x\cos x + 2e^x\sin x \\ y' = c_2 - c_3 e^{-x} - \cancel{e^x\cos x} + 3e^x\sin x + \cancel{2}e^x\cos x \\ y'' = c_3 e^{-x} + \cancel{3}\,2e^x\sin x + 4e^x\cos x - \cancel{e^x\sin x} \end{cases}$$

$$\Rightarrow \begin{cases} c_1 + c_3 - 1 = 0 \\ c_2 - c_3 + 1 = 0 \\ c_3 + 4 = 0 \end{cases} \Rightarrow \begin{cases} c_1 = 5 \\ c_2 = -5 \\ c_3 = -4 \end{cases} \quad (+2)$$

$$\therefore\ y = 5 - 5x - 4e^{-x} - e^x\cos x + 2e^x\sin x$$