Quiz #6 (CSE 400.001)

Wednesday, November 10, 2004

Name: _____ E-mail: _____

Dept: _____ ID No: _____

1. (10 points) Using $h = 1/2$ and $k = 2/3$, approximate the solution to the following elliptic equation

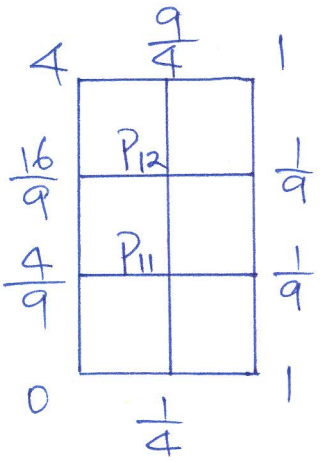
$$u_{xx} + 4u_{yy} = 9, \quad 0 < x < 1, \ 0 < y < 2$$

with boundary conditions:

$$u(x,0) = x^2, \quad u(x,2) = (x-2)^2, \quad 0 \le x \le 1;$$
$$u(0,y) = y^2, \quad u(1,y) = (y-1)^2, \quad 0 \le y \le 2.$$

Set up a system of linear equations.

i	j	x_i	y_j	$u(x_i, y_j)$
1	1	1/2	2/3	
1	2	1/2	4/3	



$$\frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} + 4 \cdot \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} = 9 \quad (+3)$$

$$4(u_{i+1,j} - 2u_{i,j} + u_{i-1,j}) + 9(u_{i,j+1} - 2u_{i,j} + u_{i,j-1}) = 9 \quad (+2)$$

$$4u_{i+1,j} - 26u_{i,j} + 4u_{i-1,j} + 9u_{i,j+1} + 9u_{i,j-1} = 9 \quad (+1)$$

$$\begin{cases} P_{11}: & -26u_{11} + 9u_{12} = \dfrac{163}{36} \quad (+2) \\ P_{12}: & -26u_{12} + 9u_{11} = -\dfrac{677}{36} \quad (+2) \end{cases}$$

2. (15 points) Consider the following hyperbolic equation

$$u_{tt} = u_{xx} + 100, \quad 0 \le x \le 1, \ 0 \le t \le 0.4,$$

with intial and boundary conditions

$$u(x,0) = x^3, \quad u_t(x,0) = x^2; \quad u_x(0,t) = t^2, \quad u(1,t) = (1+t)^3,$$

Approximate the solution to above equation with $h = k = 0.2$, for $0 \le t \le 0.4$.

(a) (5 points) Represent $u_{i,j+1}$ in terms of $u_{i-1,j}, u_{i,j}, u_{i+1,j}, u_{i,j-1}$.

(b) (5 points) Represent $u_{i,1}$ in terms of $u_{i-1,0}, u_{i,0}, u_{i+1,0}$.

(c) (5 points) Represent $u_{0,j+1}$ in terms of $u_{0,j}, u_{1,j}, u_{0,j-1}$.

(a) $\frac{1}{k^2}\left[u_{i,j+1} - 2u_{ij} + u_{i,j-1}\right] = \frac{1}{h^2}\left[u_{i+1,j} - 2u_{ij} + u_{i+1,j}\right] + 100$

$u_{i,j+1} = u_{i+1,j} + u_{i+1,j} - u_{i,j-1} + 4$ (+2) (+3)

(b) $u_{i,1} - u_{i,-1} = 2k(h^2 i^2) = 0.016\, i^2$ (+2)

$u_{i,1} = u_{i+1,0} + u_{i+1,0} - u_{i,1} + 0.016\, i^2 + 4$ (+2)

$u_{i,1} = \frac{1}{2}(u_{i+1,0} + u_{i+1,0}) + 0.008\, i^2 + 2$ (+1)

(c) $u_{1,j} - u_{-1,j} = 2h(k^2 j^2) = 0.016\, j^2$ (+2)

$u_{0,j+1} = u_{1,j} + u_{-1,j} - u_{0,j-1} + 4$

$= u_{1,j} + u_{1,j} - 0.016\, j^2 - u_{0,j-1} + 4$ (+3)

$= 2u_{1,j} - u_{0,j-1} - 0.016\, j^2 + 4$