Quiz #5 (CSE 400.001)

November 16, 2011 (Wednesday)

1. (10 points) Compute the following integral using the Gauss quadrature with $n = 2, 3, 4$ and compare the results with the exact result.

$$\int_0^4 (2x+1)dx$$

Solution:

Let $x = 2t + 2$, then $dx = 2dt$ and $\int_0^4 (2x+1)dx = \int_{-1}^1 (8t+10)dt = 20$

(i) $n = 2$: $\int_{-1}^1 (8t+10)dt = 1 * [8*(-0.57735)+10] + 1*[8*0.57735+10] = 20$, $\epsilon = 0$

(ii) $n = 3$: $\int_{-1}^1 (8t+10)dt = 0.55556*[8*(-0.77460)+10] + 0.88889*[10]$
$+0.55556*[8*0.77460+10] = 20.0001$, $\epsilon = 0.0001$

(iii) $n = 4$: $\int_{-1}^1 (8t+10)dt = 0.34785*[8*(-0.86113)+10]+0.65215*[8*(-0.33998)+10]$
$+0.65215*[8*0.33998+10]+0.34785*[8*0.86113+10] = 20$,
$\epsilon = 0$

2. (5 points) Find a good way to compute

$$\sqrt{x^2+100} - 10$$

for small $|x| << 1$.

Solution:

$$\sqrt{x^2+100} - 10 = \frac{x^2}{\sqrt{x^2+100}+10}$$

3. (5 points) Interpolate

$$f_0 = f(-2) = 2,\ f_1 = f(0) = 1,\ f_2 = f(2) = 8$$

by the cubic spline satisfying $k_0 = -2$ and $k_2 = 2$.

Solution:

$$k_0 + 4k_1 + k_2 = \tfrac{3}{2}\cdot(6) = 9 \implies 4k_1 = 9 \implies k_1 = \frac{9}{4}$$

$$\begin{cases} p_0(x) &= Ax^3 + Bx^2 + \frac{9}{4}x + 1, \quad \text{for } -2 \le x \le 0 \\ p_0'(x) &= 3Ax^2 + 2Bx + \frac{9}{4}, \quad \text{for } -2 \le x \le 0 \\ p_1(x) &= ax^3 + bx^2 + \frac{9}{4}x + 1, \quad \text{for } 0 \le x \le 2 \\ p_1'(x) &= 3ax^2 + 2bx + \frac{9}{4}, \quad \text{for } 0 \le x \le 2 \end{cases}$$

$$\begin{cases} p_0(-2) &= -8A + 4B - \frac{9}{2} + 1 = 2, \\ p_0'(-2) &= 12A - 4B + \frac{9}{4} = -2, \\ p_1(2) &= 8a + 4b + \frac{9}{2} + 1 = 8, \\ p_1'(2) &= 12a + 4b + \frac{9}{4} = 2. \end{cases}$$

$$\begin{cases} p_0(x) &= \frac{5}{16}x^3 + 2x^2 + \frac{9}{4}x + 1, \quad \text{for } -2 \le x \le 0 \\ p_1(x) &= -\frac{11}{16}x^3 + 2x^2 + \frac{9}{4}x + 1, \quad \text{for } 0 \le x \le 2 \end{cases}$$