Quiz #3 (CSE 400.001)

Monday, October 15, 2012

Name: _____ **E-mail:** _____

Dept: _____ **ID No:** _____

1. (10 points) Solve the following initial value problem:

$$\begin{aligned} y_1' &= -5y_1 + 2y_2, \quad y_1(0) = 3 \\ y_2' &= \ \ 2y_1 - 2y_2, \quad y_2(0) = 1 \end{aligned}$$

$A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$, $\det(A - \lambda I) = \lambda^2 + 7\lambda + 6 = 0$ (+2)

$\lambda_1 = -1$, $x^{(1)} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$; $\lambda_2 = -6$, $x^{(2)} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ (+2) (+2)

$\begin{bmatrix} y_1(x) \\ y_2(x) \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-x} + c_2 \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{-6x}$ (+2)

$\left.\begin{aligned} c_1 + 2c_2 &= 3 \\ 2c_1 - c_2 &= 1 \end{aligned}\right] \Rightarrow \quad c_1 = 1, \ c_2 = 1$

$\therefore \begin{bmatrix} y_1(x) \\ y_2(x) \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-x} + \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{-6x}$ (+2)

2. (10 points) Solve the following equation using the Power Series Method:

$$y'' - x^2 y = 0.$$

$$y = \sum_{m=0}^{\infty} a_m x^m, \quad x^2 y = \sum_{m=0}^{\infty} a_m x^{m+2} = \sum_{s=2}^{\infty} a_{s-2} x^s \quad (+1)$$

$$y'' = \sum_{m=2}^{\infty} m(m-1) a_m x^{m-2} = \sum_{s=0}^{\infty} (s+2)(s+1) a_{s+2} x^s \quad (+1)$$

$$2a_2 + 6a_3 x + \sum_{s=2}^{\infty} \left[(s+2)(s+1) a_{s+2} - a_{s-2} \right] x^s = 0$$

$(+1)$ $\qquad$ $(+1)$

$$\therefore a_2 = a_3 = 0, \quad a_{s+2} = \frac{1}{(s+2)(s+1)} a_{s-2}, \ (s = 2, 3, 4, \cdots)$$

$(+1)$

$$a_4 = \frac{1}{4 \cdot 3} a_0 = \frac{2!}{4!} a_0, \quad a_5 = \frac{1}{5 \cdot 4} a_1 = \frac{3!}{5!} a_1$$

$$a_6 = \frac{1}{6 \cdot 5} a_2 = 0, \quad a_7 = \frac{1}{7 \cdot 6} a_3 = 0$$

$$a_8 = \frac{1}{8 \cdot 7} a_4 = \frac{6! 2!}{8! 4!} a_0, \quad a_9 = \frac{1}{9 \cdot 8} a_5 = \frac{7! 3!}{9! 5!} a_1$$

$$\cdots$$

$$\therefore y = \underbrace{a_0 + a_1 x}_{(+1)} + a_0 \left[\frac{2!}{4!} x^4 + \frac{6! 2!}{8! 4!} x^8 + \cdots \right] \ (+2)$$

$$+ a_1 \left[\frac{3!}{5!} x^5 + \frac{7! 3!}{9! 5!} x^9 + \cdots \right] \ (+2)$$