Quiz #6 (CSE 400.001)

Thursday, May 31, 2001

Name: ______________________ **E-mail:** ______________________

Dept: ______________________ **ID No:** ______________________

1. (7 points) Apply the classical Runge-Kutta method with $h = 0.1$ to the initial value problem:

$$y' = 2 - 2y, \quad y(0) = 0.$$

Represent y_{n+1} in terms of x_n and y_n.

$$f(x, y) = 2 - 2y$$

$$k_1 = 0.1 * [2 - 2y_n] = 0.2 - 0.2y_n$$

$$\begin{aligned} k_2 &= 0.1 * [2 - 2(y_n + 0.1 - 0.1y_n)] \\ &= 0.2 - 0.2(0.9y_n + 0.1) \\ &= 0.18 - 0.18y_n \end{aligned}$$

$$\begin{aligned} k_3 &= 0.1 * [2 - 2(y_n + 0.09 - 0.09y_n)] \\ &= 0.2 - 0.2(0.91y_n + 0.09) \\ &= 0.182 - 0.182y_n \end{aligned}$$

$$\begin{aligned} k_4 &= 0.1 * [2 - 2(y_n + 0.182 - 0.182y_n)] \\ &= 0.2 - 0.2(0.818y_n + 0.182) \\ &= 0.1636 - 0.1636y_n \end{aligned}$$

$$\begin{aligned} y_{n+1} &= y_n + \frac{1}{6}[0.2 + 0.36 + 0.364 + 0.1636 - 0.2y_n - 0.36y_n - 0.364y_n - 0.1636y_n] \\ &\approx y_n + [0.181267 - 0.181267y_n] \\ &= \mathbf{0.818733y_n + 0.181267} \end{aligned}$$

2001년 Midterm II

5. **(20 points)** Table 1 compares the results of applying the Euler, improved Euler, and Runge-Kutta methods to the following initial value problem with $h = 0.05$:

$$y' = 2xy, \quad y(1) = 1.$$

Fill in the three blanks (A), (B), (C); and show your work for partial credit.

(A) Euler Method : +4

$$y_{n+1} = y_n + h f(x_n, y_n)$$
$$= y_n + 0.05 * 2 * x_n * y_n$$

(B) Improved Euler Method

k_1 : +2

k_2 : +2

$$y_{n+1} = y_n + \frac{1}{2}[k_1 + k_2] \quad +2$$

(C) Runge-Kutta Method

k_1, k_2, k_3, k_4 각 +2 × 4 = 8점

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \quad +2$$

Table 1

Comparison of numerical methods with $h = 0.05$

x_n	Euler	Improved Euler	Runge–Kutta	True value
1.00	1.0000	1.0000	1.0000	1.0000
1.05	1.1000	1.1077	1.1079	1.1079
1.10	1.2155	1.2332	1.2337	1.2337
1.15	1.3492	1.3798	1.3806	1.3806
1.20	1.5044	1.5514	1.5527	1.5527
1.25	1.6849	1.7531	1.7551	1.7551
1.30	1.8955	1.9909	1.9937	1.9937
1.35	2.1419	2.2721	2.2762	2.2762
1.40	2.4311	2.6060	2.6117	2.6117
1.45	2.7714	3.0038	3.0117	3.0117
1.50	3.1733	3.4795	3.4903	3.4904

2002년 Midterm II

5. (15 points) Table 1 shows the result of applying the Improved Euler method to the following initial value problem with $h = 0.25$:

$$y' = 1 + y/x, \quad \text{for } 1 \le x \le 2, \quad y(1) = 2.$$

Fill in the blank; and show your work for partial credit.

x_i	y_i
1.25	2.7750000
1.50	3.60083
1.75	4.4688294
2.00	5.3728586

Table 1: Improved Euler Method

$x_1 = 1.25, \quad y_1 = 2.775$

$h = 0.25, \quad f(x, y) = 1 + \frac{y}{x}$ (+2)

$k_1 = 0.25 * f(x_1, y_1)$

$= 0.25 * \left(1 + \frac{2.775}{1.25}\right)$

$= 0.805$ (+5)

$k_2 = 0.25 * f(x_2, y_1 + k_1)$

$= 0.25 * \left(1 + \frac{2.775 + 0.805}{1.5}\right)$

$= 0.846667$ (+5)

$y_2 = y_1 + \frac{1}{2}(k_1 + k_2)$

$= 2.775 + \frac{1}{2}(0.805 + 0.846667)$

$= 3.60083$ (+3)

2002년 Midterm II

6. (20 points) Table 2 shows the result of applying the Runge-Kutta method to the following initial value problem with $h = 0.2$:

$$y' = y - x^2 + 1, \quad \text{for } 0 \le x \le 1, \quad y(0) = 0.5.$$

Fill in the blank; and show your work for partial credit.

x_i	y_i
0.2	0.8292933
0.4	1.21408
0.6	1.6489220
0.8	2.1272027
1.0	2.6408227

Table 2: Runge-Kutta Method

$x_1 = 0.2,\ y_1 = 0.8292933$

$h = 0.2,\ f(x, y) = 1 - x^2 + y$ +2

$k_1 = 0.2 * f(x_1, y_1) = 0.2 * (1 - 0.2^2 + 0.8292933)$ +4

$= 0.357859$

$k_2 = 0.2 * f(x_1 + 0.1, y_1 + 0.5k_1)$

$= 0.2 * (1 - 0.3^2 + 0.8292933 + 0.178930)$ +4

$= 0.383645$

$k_3 = 0.2 * f(0.3, y_1 + 0.5k_2)$ +4

$= 0.2 * (1 - 0.3^2 + 0.8292933 + 0.191823)$

$= 0.386223$

$k_4 = 0.2 * f(x_2, y_1 + k_3) = \cancel{0.2 * (1 - 0.4^2 +}$ +4

$= 0.2 * (1 - 0.4^2 + 0.8292933 + 0.386223)$

$= 0.411103$

$y_2 = y_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = 1.21408$ +2

2003년_Midterm II

3. (15 points) Table 1 shows the result of applying the Runge-Kutta method to the following initial value problem with $h = 0.2$:

$$y' = -(y+1)(y+3), \quad \text{for } 0 \le x \le 1, \quad y(0) = -2.$$

Fill in the blank; and show your work for partial credit.

x_i	y_i
0.2	-1.80263
0.4	
0.6	-1.46296
0.8	-1.33598
1.0	-1.23843

Table 1: Runge-Kutta Method

$x_1 = 0.2, \quad y_1 = -1.80263$
$h = 0.2, \quad f(x,y) = -(y+1)(y+3)$ (+2)

$k_1 = 0.2 * f(x_1, y_1) = 0.19221$ (+2)

$k_2 = 0.2 * f(x_1 + 0.1, y_1 + 0.5k_1) = 0.18277$ (+3)

$k_3 = 0.2 * f(x_1 + 0.1, y_1 + 0.5k_2) = 0.18332$ (+3)

$k_4 = 0.2 * f(x_2, y_1 + k_3) = 0.17101$ (+3)

$y_2 = y_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = -1.62006$ (+2)

2004년 Midterm II

4. (10 points) Table 1 shows the result of applying the Improved Euler method to the following initial value problem with $h = 0.2$:

$$y' = -(y+1)(y+3), \quad \text{for } 0 \le x \le 1, \quad y(0) = -2.$$

Fill in the blank; and show your work for partial credit.

x_i	y_i
0.2	-1.80400
0.4	-1.62292
0.6	-1.46724
0.8	-1.34132
1.0	-1.24429

Table 1: Improved Euler Method

$x_1 = 0.2, \quad y_1 = -1.80400$

$h = 0.2, \quad f(x, y) = -(y+1)(y+3)$ +2

$k_1 = 0.2 * f(x_1, y_1) = 0.192317$ +2

$k_2 = 0.2 * f(x_1 + 0.2, y_1 + k_1) = 0.169842$ +3

$y_2 = y_1 + \frac{1}{2}(k_1 + k_2) = -1.62292$ +2