Quiz #5 (CSE 400.001)

Monday, December 8, 2014

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1. (10 points) Show that two matrices $A_{m\times n}$ and $B_{n\times r}$ can be multiplied as follows:

$$AB = \sum_{k=1}^{n} (\text{column } k \text{ of } A)(\text{row } k \text{ of } B)$$

$$A = \begin{bmatrix} \boxed{\mathbb{a}_1} & \cdots & \boxed{\mathbb{a}_n} \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} \boxed{\mathbb{b}_1^T} \\ \vdots \\ \boxed{\mathbb{b}_n^T} \end{bmatrix}$$

$$\mathbb{a}_k \cdot \mathbb{b}_k^T = \begin{bmatrix} a_{1k} \\ \vdots \\ a_{mk} \end{bmatrix} \begin{bmatrix} b_{k1} & \cdots & b_{kr} \end{bmatrix} = \begin{bmatrix} a_{1k}b_{k1} & \cdots & a_{1k}b_{kr} \\ \vdots & & \vdots \\ a_{mk}b_{k1} & \cdots & a_{mk}b_{kr} \end{bmatrix}$$

$$(\mathbb{a}_k \cdot \mathbb{b}_k^T)_{ij} = a_{ik}b_{kj}$$

$$\left(\sum_{k=1}^{n} \mathbb{a}_k \cdot \mathbb{b}_k^T\right)_{ij} = \sum_{k=1}^{n} (\mathbb{a}_k \cdot \mathbb{b}_k^T)_{ij} = \sum_{k=1}^{n} a_{ik} \cdot b_{kj}$$

$$= (\text{row } i \text{ of } A) \cdot (\text{column } j \text{ of } B)$$

$$= (AB)_{ij}$$

$$\therefore AB = \sum_{k=1}^{n} \mathbb{a}_k \cdot \mathbb{b}_k^T$$

2. (10 points) If an invertible matrix $A = L_1 D_1 U_1 = L_2 D_2 U_2$, show that the factorization is unique: $L_1 = L_2$, $D_1 = D_2$, and $U_1 = U_2$.

$$L_1 D_1 U_1 = L_2 D_2 U_2$$

$$\underbrace{L_2^{-1} L_1}_{\text{Lower Triangular with 1s on the diagonal}} = \underbrace{D_2 U_2 U_1^{-1} D_1^{-1}}_{\text{Upper Triangular}}$$

$\therefore L_2^{-1} L_1 = I$ and $L_1 = L_2$

$$\Rightarrow \quad D_1 U_1 = D_2 U_2$$

$$\underbrace{D_2^{-1} D_1}_{\text{Diagonal}} = \underbrace{U_2 U_1^{-1}}_{\text{Upper Triangular with 1s on the diagonal}}$$

$$\Rightarrow \quad D_2^{-1} D_1 = I, \quad U_2 U_1^{-1} = I$$

$$\therefore D_1 = D_2, \quad U_1 = U_2$$

$$\Rightarrow \quad L_1 = L_2, \quad D_1 = D_2, \quad U_1 = U_2$$