Quiz #2 (EngMath I) [Wednesday, Sept. 23, 2015]

Name: ______________________ ID No: ______________________

1. (10 points) Find a particular solution of the following differential equation:

$$y'''' + 4y''' + 6y'' + 4y' + y = x^3e^{-x}.$$

Hint: Make the substitution $y(x) = e^{-x}u(x)$ and solve for $u(x)$.

$$y(x) = e^{-x} \cdot u(x)$$

$$y'(x) = -e^{-x} \cdot u(x) + e^{-x} \cdot u'(x)$$ +4

$$y''(x) = e^{-x} \cdot u(x) - 2e^{-x} u'(x) + e^{-x} \cdot u''(x)$$

$$y'''(x) = -e^{-x} \cdot u(x) + 3e^{-x} \cdot u'(x) - 3e^{-x} u''(x) + e^{-x} u'''(x)$$

$$y''''(x) = e^{-x} u(x) - 4e^{-x} u'(x) + 6e^{-x} u''(x) - 4e^{-x} u'''(x) + e^{-x} u''''(x)$$

$$y'''' + 4y''' + 6y'' + 4y' + y = e^{-x} \cdot u''''(x) = x^3 e^{-x}$$

$$u''''(x) = x^3$$ +3

$$u_p(x) = \frac{1}{840} x^7$$ +2

$$\therefore y_p(x) = e^{-x} \cdot u_p(x) = \frac{1}{840} x^7 \cdot e^{-x}$$ +1

2. (15 points) Using the substitution $x = e^t$, solve the following differential equation:

$$2x^2y'' + xy' - y = 3x - 5x^2.$$

$$\hat{y}(t) = y(e^t) = y(x)$$
$$\hat{y}'(t) = y'(e^t)\cdot e^t = x\cdot y'(x)$$
$$\hat{y}''(t) = y''(e^t)\cdot e^{2t} + y'(e^t)\cdot e^t$$
$$= x^2 y''(x) + \hat{y}'(t)$$

+5

$$2(\hat{y}''(t) - \hat{y}'(t)) + \hat{y}'(t) - \hat{y}(t) = 3e^t - 5e^{2t}$$
$$2\hat{y}''(t) - \hat{y}'(t) - \hat{y}(t) = 3e^t - 5e^{2t}$$

+3

$$2\lambda^2 - \lambda - 1 = (\lambda - 1)(2\lambda + 1) = 0$$
$$\therefore \hat{y}_h(t) = c_1 e^t + c_2 e^{-\frac{1}{2}t}$$

+1

$$\hat{y}_p(t) = Ate^t + Be^{2t}$$ +2

$$2\hat{y}_p''(t) - \hat{y}'(t) - \hat{y}(t) = 3Ae^t + 5Be^{2t}$$
$$= 3e^t - 5e^{2t}$$
$$\therefore A = 1, \quad B = -1$$

+2

$$\therefore y(x) = \hat{y}(t) = c_1 e^t + c_2 e^{-\frac{1}{2}t} + te^t - e^{2t}$$
$$= c_1 x + c_2 \frac{1}{\sqrt{x}} + x\ln|x| - x^2$$

+2