Engineering Mathematics I

Midterm Exam, October 14, 2015

Problem	Score
1	
2	
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Total	

Name: _____

ID No: _____

Dept: _____

E-mail: _____

1. (25 points) Newton's law of cooling states that the time rate of change of the temperature $T(t)$ of a cooling body is proportional to the temperature difference between the body and its surroundings: $dT/dt = k(T - S)$, where $S(t)$ is the temperature of the surroundings and k is a constant.

   (a) (10 points) A body at a temperature of 100° is placed in a room of unknown temperature. The room temperature does not change. If after 10 min the body has cooled to 90° and after 20 min to 85°, find the temperature of the surroundings.

   (b) (15 points) A body of temperature 100° is placed in water of temperature 50°. After 10 min the temperature of the body is 80° and the temperature of the water is 60°. Assuming all the heat lost by the body is absorbed by the water: $S(t) - S(0) = c(T(t) - T(0))$, for some constant c, find the temperature of the body and of the water at any time. Find the equilibrium temperature.

2. (15 points) Using the method of variation of parameters, solve the following system of equations:

$$\begin{aligned} y_1' &= -2y_1 + y_2 + 2e^{-t} \\ y_2' &= y_1 - 2y_2 + 3t \end{aligned}$$

3. (10 points) Solve the following initial value problem by the power series method. Find the recurrence formula and find the first six nonzero terms in the series.

$$y'' + x^2 y = 0, \quad y(0) = 12,\ y'(0) = 20.$$

4. (15 points) Using Laplace transforms, solve the following initial value problem:

$$y'' - 3y' + 2y = e^{-t}, \quad y(1) = 1,\ y'(1) = 0.$$

5. (15 points) Using Laplace transforms, show that

$$\int_0^x \left[\int_0^t f(u)\ du \right] dt = \int_0^x f(t)(x-t)\ dt$$

6. (10 points) Find a function $f(t)$, if it exists. Otherwise, explain why there is no such solution.

   (a) (3 points) $t * f(t) = t^4$

   (b) (3 points) $1 * 1 * f(t) = \frac{1}{2}t^2$

   (c) (4 points) $1 * f(t) = 1$

7. (10 points) Using Laplace transforms, solve the following initial value problem:

$$y'' + 2y' + 2y = \delta(t - \pi), \quad y(0) = 0,\ y'(0) = 0.$$