

Geometric Modeling

(CSE 4190.667)

Midterm Exam: April 29, 2015

Problem	Score
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Total	

Name: _____

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1. (15 points) What is the cubic Bézier form of the following 3D curve segment defined for $t \in [0, 1]$?

$$\mathbf{x}(t) = \begin{bmatrix} 2t + 1 \\ t^2 + 3 \\ (t + 1)^3 \end{bmatrix}.$$

2. (10 points) Degree elevate the following quadratic curve to a cubic Bézier curve by computing the four control points \mathbf{b}_i , for $i = 0, 1, 2, 3$:

$$\mathbf{x}(t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} (1 - t)^2 + \begin{bmatrix} 1 \\ 0 \end{bmatrix} 2(1 - t)t + \begin{bmatrix} 2 \\ 3 \end{bmatrix} t^2, \quad \text{for } 0 \leq t \leq 1.$$

3. (20 points) Using the 3-stage de Casteljau evaluation method, compute $\mathbf{x}_u(0.5, 0.5)$, $\mathbf{x}_v(0.5, 0.5)$, and $\mathbf{n}(0.5, 0.5)$ for the Bézier patch defined by the following control points:

$$\begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 2 \\ 2 \end{bmatrix} & \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 2 \\ 2 \end{bmatrix} & \begin{bmatrix} 2 \\ 0 \\ 0 \\ 2 \\ 1 \\ 1 \\ 2 \\ 2 \\ 2 \end{bmatrix} & \begin{bmatrix} 3 \\ 0 \\ 0 \\ 3 \\ 1 \\ 1 \\ 3 \\ 2 \\ 2 \end{bmatrix} \end{bmatrix}$$

4. (20 points) Given a knot sequence $0, 0, 1, 2, 3, 3$ for a quadratic B-spline curve:

$$\mathbf{x}(u) = (2u, N_2^2(u) + 3N_3^2(u)), \quad 0 \leq u \leq 3,$$

- (a) (10 points) What are the B-spline control points \mathbf{d}_i for the quadratic curve $\mathbf{x}(u)$?
- (b) (10 points) Inserting two additional knots 1 and 2, the B-spline curve $\mathbf{x}(u)$ can be converted to a connected sequence of three quadratic Bézier curves. What are their control points?

5. (20 points) Find the cubic spline interpolant with a knot sequence $0, 0, 0, 1, 4, 4, 4$ for the following data set and derivative vectors:

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 12 \\ 0 \end{bmatrix}, \quad \text{and} \quad \mathbf{t}_s = \begin{bmatrix} -3 \\ 3 \end{bmatrix}, \quad \mathbf{t}_e = \begin{bmatrix} 3 \\ 3 \end{bmatrix}.$$

6. (15 points) Let a rational cubic Bézier generatrix be given by four control points

$$\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix},$$

and weights 1, 2, 2, 1. What are the Bézier control points and weights of the first quadrant of the corresponding surface of revolution?