Programming #I-2 (4190.667)

Due: April 1, 2015

Part I: Design an interactive system that can control the shape of two cubic Bézier curves:

$$C(u) = \sum_{i=0}^{3} \mathbf{p}_i B_i^3(u), \quad D(v) = \sum_{j=0}^{3} \mathbf{q}_j B_j^3(v), \quad 0 \le u, v \le 1,$$

by dragging their control points. Moreover, using the subdivision algorithm discussed in the textbook, implement an algorithm for computing the intersection points between the two curves and the self-intersection points of each curve. Display the AABBs that are generated in the search for the intersection points.

Part II: Let $L_{[a,b]}(t)$, $(0 \le a \le t \le b \le 1)$, denote a line segment connecting two curve points $C(a)$ and $C(b)$. The condition of Filip et al. (CAGD, 1986) implies that, for $a \le t \le b$,

$$\begin{aligned}
\|C(t) - L_{[a,b]}(t)\| &\le \frac{(b-a)^2}{8} \max_{a \le t \le b} \|C''(t)\| \\
&\le \frac{(b-a)^2}{8} \max_{0 \le t \le 1} \|C''(t)\| \\
&\le \frac{3}{4} \cdot (b-a)^2 \cdot \max\{\|\mathbf{p}_0 - 2\mathbf{p}_1 + \mathbf{p}_2\|, \|\mathbf{p}_1 - 2\mathbf{p}_2 + \mathbf{p}_3\|\} = \epsilon_{b-a}.
\end{aligned}$$

This means that, when a cubic Bézier curve is subdivided into two halves, the error between each curve segment and the line connecting the endpoints is reduced 4 times. Now, expanding the AABB (with $C(a)$ and $C(b)$ as two opposite corners) outwards by ϵ_{b-a}, we can bound the subcurve segment $C(t), (a \le t \le b)$, within this expanded AABB. Reimplement the curve-curve intersection algorithm using the new AABB construction scheme.