

Programming #I-3 (4190.667)

Due: April 8, 2015

Part I: Design an interactive system that can control the shape of two cubic Bézier curves:

$$C(u) = \sum_{i=0}^3 \mathbf{p}_i B_i^3(u), \quad D(v) = \sum_{j=0}^3 \mathbf{q}_j B_j^3(v), \quad 0 \leq u, v \leq 1,$$

by dragging their control points. Moreover, using the subdivision algorithm discussed in the textbook, implement an algorithm for computing the intersection points between the two curves and the self-intersection points of each curve. Display the AABBs that are generated in the search for the intersection points.

Part II: Let $L_{[a,b]}(t)$, ($0 \leq a \leq t \leq b \leq 1$), denote a line segment connecting two curve points $C(a)$ and $C(b)$. The condition of Filip et al. (CAGD, 1986) implies that, for $a \leq t \leq b$,

$$\begin{aligned} \|C(t) - L_{[a,b]}(t)\| &\leq \frac{(b-a)^2}{8} \max_{a \leq t \leq b} \|C'''(t)\| \\ &\leq \frac{(b-a)^2}{8} \max_{0 \leq t \leq 1} \|C'''(t)\| \\ &\leq \frac{3}{4} \cdot (b-a)^2 \cdot \max\{\|\mathbf{p}_0 - 2\mathbf{p}_1 + \mathbf{p}_2\|, \|\mathbf{p}_1 - 2\mathbf{p}_2 + \mathbf{p}_3\|\} = \epsilon_{b-a}. \end{aligned}$$

This means that, when a cubic Bézier curve is subdivided into two halves, the error between each curve segment and the line connecting the endpoints is reduced 4 times. Now, expanding the AABB (with $C(a)$ and $C(b)$ as two opposite corners) outwards by ϵ_{b-a} , we can bound the subcurve segment $C(t)$, ($a \leq t \leq b$), within this expanded AABB. Reimplement the curve-curve intersection algorithm using the new AABB construction scheme.

Part III: Line swept sphere (LSS) is a volume generated by sweeping a ball along a line segment. Using the LSS bounding volume (generated by sweeping a disk of radius ϵ_{b-a} along the line segment $\overline{C(a)C(b)}$), we can bound the curve segment $C(t)$, ($a \leq t \leq b$), more tightly than the AABBs discussed above for Parts I and II. Reimplement the curve-curve intersection algorithm using the LSS construction scheme.