Computer Graphics

(Comp 4190.410)

Midterm Exam: October 26, 2016

Problem	Score
1	
2	
3	
4	
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6	
Total	

Name: _____

ID No: _____

Dept: _____

1. (10 points) Consider a rotation $R_x(120^\circ)$ followed by another rotation $R_y(120^\circ)$. What are the rotation axis and the rotation angle for the composite rotation?

2. (10 points) Consider a trackball of radius 1 with its center located at the origin $(0, 0, 0)$. A sensor is installed to the housing of the trackball at a location $\mathbf{p} = (p_x, p_y, p_z) \in S^2$, where the sensor can detect a 2D surface velocity $\mathbf{p}' = (p'_x, p'_y, p'_z)$ of the rotating trackball. (Note that the surface velocity $\mathbf{p}'$ is orthogonal to $\mathbf{p}$, i.e., $\langle \mathbf{p}, \mathbf{p}' \rangle = 0$.) There are certain angular velocities $\omega = (\omega_x, \omega_y, \omega_z)$ that cannot be detected by this sensor. Discuss what are the corresponding 3D rotations using the following relation:

$$\begin{bmatrix} 0 & p_z & -p_y \\ -p_z & 0 & p_x \\ p_y & -p_x & 0 \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} p'_x \\ p'_y \\ p'_z \end{bmatrix}$$

3. (20 points) Let $C(t)$ be a cubic Bézier curve given by four control points:

$$\mathbf{b}_0 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \quad \mathbf{b}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \quad \mathbf{b}_3 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

Subdivide the curve $C(t)$ at x-extreme points, y-extreme points and inflection points, and compute the control points for each inflection-free x, y-monotone curve subsegment.

4. (20 points) Design an efficient algorithm for computing the minimum distance between two circular arcs. You may assume that the circular arcs are x and y-monotone segments.

5. (20 points) Discuss how to extend your algorithm for computing the minimum distance from a point to a curve/surface to a more general algorithm for computing the minimum distance between two curves/surfaces.