Programming #1: Part I (4190.410)

Due: September 21, 2016

Given a cubic Bézier curve $C(t) = \sum_{l=0}^{3} \mathbf{b}_l B_l^3(t)$, $0 \le t \le 1$, with four control points $\mathbf{b}_0, \mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3$, we have the following differential properties:

$$
\begin{aligned}
C(t) = (x(t), y(t)) &= (1-t)^3\ \mathbf{b}_0 + 3(1-t)^2 t\ \mathbf{b}_1 + 3(1-t)t^2\ \mathbf{b}_2 + t^3\ \mathbf{b}_3, \\
C'(t)/3 = (x'(t), y'(t))/3 &= (1-t)^2\ (\mathbf{b}_1 - \mathbf{b}_0) + 2(1-t)t\ (\mathbf{b}_2 - \mathbf{b}_1) + t^2\ (\mathbf{b}_3 - \mathbf{b}_2) \\
&= (\mathbf{b}_1 - \mathbf{b}_0) + t\ (2\mathbf{b}_2 - 4\mathbf{b}_1 + 2\mathbf{b}_0) + t^2\ (\mathbf{b}_3 - 3\mathbf{b}_2 + 3\mathbf{b}_1 - \mathbf{b}_0), \\
C''(t)/6 = (x''(t), y''(t))/6 &= (1-t)\ (\mathbf{b}_2 - 2\mathbf{b}_1 + \mathbf{b}_0) + t\ (\mathbf{b}_3 - 2\mathbf{b}_2 + \mathbf{b}_1) \\
&= (\mathbf{b}_2 - 2\mathbf{b}_1 + \mathbf{b}_0) + t\ (\mathbf{b}_3 - 3\mathbf{b}_2 + 3\mathbf{b}_1 - \mathbf{b}_0).
\end{aligned}
$$

Part I: Compute all x and y-extreme points as well as all inflection points of $C(t)$, $0 \le t \le 1$, by solving the following three quadratic equations:

$$x'(t)/3 = 0, \quad y'(t)/3 = 0, \quad (x'(t)y''(t) - x''(t)y'(t))/18 = 0.$$

Moreover, compute all curvature extreme points by solving the following quintic equation:

$$a'(t)b(t) - 3a(t)(b'(t)/2) = 0,$$

where $a(t) = x'(t)y''(t) - x''(t)y'(t)$ and $b(t) = x'(t)^2 + y'(t)^2$.