Programming #2: Part I (4190.410)

Due: October 19, 2016

A bicubic Bézier surface $S(u,v) = \sum_{k=0}^{3}\sum_{l=0}^{3} \mathbf{b}_{kl} B_k^3(u) B_l^3(v)$, $0 \le u, v \le 1$, can be bounded by a hierarchy of unions of Tetrahedron Swept Spheres $O_{\epsilon_h}(T_{ij}^h)$, each bounding the surface patch $S_{ij}^h(u,v) = S(u,v)$, where $(i-1)/2^h \le u \le i/2^h$, $(j-1)/2^h \le v \le j/2^h$, for $i, j = 1, \cdots, 2^h$. The tetrahedron T_{ij}^h is determined by the four corners of $S_{ij}^h(u,v)$: $S((i-1)/2^h, (j-1)/2^h)$, $S((i-1)/2^h, j/2^h)$, $S(i/2^h, (j-1)/2^h)$, and $S(i/2^h, j/2^h)$. The radius ϵ_{ij}^h can be taken as

$$\begin{aligned}\epsilon_{ij}^h \;=\; \frac{1}{2^{2h+3}} \cdot \max\{ & 6 \max_{k=1,2;0\le l\le 3} \|\mathbf{b}_{k+1,l} - 2\mathbf{b}_{k,l} + \mathbf{b}_{k-1,l}\|, \\ & 6 \max_{0\le k\le 3;l=1,2} \|\mathbf{b}_{k,l+1} - 2\mathbf{b}_{k,l} + \mathbf{b}_{k,l-1}\|, \\ & 18 \max_{k,l=0,1,2} \|\mathbf{b}_{k+1,l+1} - \mathbf{b}_{k+1,l} - \mathbf{b}_{k,l+1} + \mathbf{b}_{k,l}\|\}\end{aligned}$$

Part I: Design an interactive system that can control the position of a query point $\mathbf{Q}$ and the shape of $S(u,v)$ by dragging the control points $\mathbf{b}_{kl}$. Moreover, implement an algorithm for computing the projection line from $\mathbf{Q}$ to the nearest point on the surface $S(u,v)$. Display the tetrahedra that have been used in the search for the nearest point $S(\hat{u}, \hat{v})$ by the algorithm.