

Programming #2: Part I (4190.410)

Due: October 31, 2016

A bicubic Bézier surface $S(u, v) = \sum_{k=0}^3 \sum_{l=0}^3 \mathbf{b}_{kl} B_k^3(u) B_l^3(v)$, $0 \leq u, v \leq 1$, can be bounded by a hierarchy of unions of Tetrahedron Swept Spheres $O_{\epsilon_h}(T_{ij}^h)$, each bounding the surface patch $S_{ij}^h(u, v) = S(u, v)$, where $(i-1)/2^h \leq u \leq i/2^h$, $(j-1)/2^h \leq v \leq j/2^h$, for $i, j = 1, \dots, 2^h$. The tetrahedron T_{ij}^h is determined by the four corners of $S_{ij}^h(u, v)$: $S((i-1)/2^h, (j-1)/2^h)$, $S((i-1)/2^h, j/2^h)$, $S(i/2^h, (j-1)/2^h)$, and $S(i/2^h, j/2^h)$. The radius ϵ_{ij}^h can be taken as

$$\epsilon_{ij}^h = \frac{1}{2^{2h+3}} \cdot \max \left\{ \begin{aligned} &6 \max_{k=1,2; 0 \leq l \leq 3} \|\mathbf{b}_{k+1,l} - 2\mathbf{b}_{k,l} + \mathbf{b}_{k-1,l}\|, \\ &6 \max_{0 \leq k \leq 3; l=1,2} \|\mathbf{b}_{k,l+1} - 2\mathbf{b}_{k,l} + \mathbf{b}_{k,l-1}\|, \\ &18 \max_{k,l=0,1,2} \|\mathbf{b}_{k+1,l+1} - \mathbf{b}_{k+1,l} - \mathbf{b}_{k,l+1} + \mathbf{b}_{k,l}\| \end{aligned} \right\}$$

Part I: Design an interactive system that can control the position of a query point \mathbf{Q} and the shape of $S(u, v)$ by dragging the control points \mathbf{b}_{kl} . Moreover, implement an algorithm for computing the projection line from \mathbf{Q} to the nearest point on the surface $S(u, v)$. Display the tetrahedra that have been used in the search for the nearest point $S(\hat{u}, \hat{v})$ by the algorithm.

Part II: Implement a slicing algorithm for computing the intersection curves of the surface $S(u, v)$ with parallel planes: $z = c_i$, for ten different height values of c_i , ($i = 1, \dots, 10$).