Quiz #3 (EngMath I) [Wednesday, November 2, 2016]

Name: ______________ Dept: ______________ ID No: ______________

1. (10 points) Compute the Fourier series of the following function:

$$f(x+2\pi) = f(x) = \begin{cases} 0, & \text{if } -\pi < x < 0 \\ \pi - x, & \text{if } 0 < x < \pi \end{cases}$$

$$a_0 = \frac{1}{2\pi}\int_0^\pi (\pi - x)\,dx = \frac{1}{2\pi}\left[\pi x - \frac{1}{2}x^2\right]_0^\pi = \frac{1}{4}\pi \quad +2$$

$$a_n = \frac{1}{\pi}\int_0^\pi (\pi - x)\cos nx\,dx$$

$$= \frac{1}{\pi}\left[\frac{1}{n}(\pi - x)\sin nx\right]_0^\pi + \frac{1}{n\pi}\int_0^\pi \sin nx\,dx$$

$$= \frac{1}{n\pi}\left[\frac{(-1)}{n}\cos nx\right]_0^\pi$$

$$= \frac{1}{n^2\pi}\left[1 - (-1)^n\right] \quad +3$$

$$b_n = \frac{1}{\pi}\int_0^\pi (\pi - x)\sin nx\,dx$$

$$= \frac{1}{\pi}\left[-\frac{1}{n}(\pi - x)\cos nx\right]_0^\pi - \frac{1}{\pi}\int_0^\pi \frac{1}{n}\cos nx\,dx$$

$$= \frac{1}{n\pi}\left[(x - \pi)\cos nx\right]_0^\pi - \frac{1}{n^2\pi}\left[\sin nx\right]_0^\pi$$

$$= \frac{1}{n} \quad +2$$

$$f(x) = \frac{\pi}{4} + \sum_{n=1}^{\infty}\left[\frac{1-(-1)^n}{n^2\pi}\cos nx + \frac{1}{n}\sin nx\right] \quad +3$$

2. (15 points)

(a) (7 points) Compute the Fourier integral of the following function:

$$f(x) = \begin{cases} 0, & \text{if } x < 0 \\ 1, & \text{if } 0 < x < 2 \\ 0, & \text{if } x > 2 \end{cases}$$

(b) (8 points) Show that

$$\int_0^\infty \frac{\sin 2x}{x}\, dx = \frac{\pi}{2}$$

(a) $A(w) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\cos wx\, dx$

$= \frac{1}{\pi}\int_0^2 \cos wx\, dx$

$= \left[\frac{1}{w\pi}\sin wx\right]_0^2$

$= \frac{\sin 2w}{w\pi}$ +2

+1

$B(w) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\sin wx\, dx$

$= \frac{1}{\pi}\int_0^2 \sin wx\, dx$

$= -\frac{1}{w\pi}\left[\cos wx\right]_0^2$

$= \frac{1 - \cos 2w}{w\pi}$ +2

$\therefore f(x) = \int_0^\infty (A(w)\cos wx + B(w)\sin wx)\, dw$

$= \int_0^\infty \frac{\sin 2w \cos wx + (1-\cos 2w)\sin wx}{w\pi}\, dw$

$= \int_0^\infty \frac{\sin(2w - wx) + \sin wx}{w\pi}\, dw$ +2

(b) $f(x) = \frac{1}{\pi}\int_0^\infty \left[\frac{\sin(2w - wx) + \sin wx}{w}\right] dw$ +2

let $x = 2$ or $x = 0$ +4

$f(2) = \frac{1}{\pi}\int_0^\infty \frac{\sin 2w}{w}\, dw$ or $f(0) = \frac{1}{\pi}\int_0^\infty \frac{\sin 2w}{w}\, dw$

$\frac{1+0}{2} = \frac{1}{\pi}\int_0^\infty \frac{\sin 2w}{w}\, dw$ +2

$\therefore \int_0^\infty \frac{\sin 2x}{x}\, dx = \frac{\pi}{2}$