Quiz #5 (EngMath I) Wednesday, Nov. 30, 2016]

Name: ____________ **Dept:** ____________ **ID No:** ____________

1. (10 points) How could you factor A into a product UL, upper triangular times lower triangular?

when $P = \begin{bmatrix} & & 1 \\ & \cdot^{\cdot^{\cdot}} & \\ 1 & & \end{bmatrix}$

$PAP = \bar{L}\bar{U}$

$A = P\bar{L}\bar{U}P$

$= P\bar{L}PP\bar{U}P$

$= UL$

2. (5 points) Write down all six of the 3 by 3 permutation matrices, including $P = I$. Identify their inverses, which are also permutation matrices. The inverses satisfy $PP^{-1} = I$ and are on the same list.

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{\text{inverse}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \xrightarrow{\text{inverse}} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

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3. (5 points) Suppose A commutes with every 2 by 2 matrix ($AB = BA$). Show that $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a multiple of the identity, i.e., $A = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$

let $B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$AB_1 = B_1A \Rightarrow \begin{bmatrix} a & 0 \\ c & 0 \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} \Rightarrow b=0,\ c=0$

let $B_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$AB_2 = B_2A \Rightarrow \begin{bmatrix} 0 & a \\ 0 & c \end{bmatrix} = \begin{bmatrix} c & d \\ 0 & 0 \end{bmatrix} \Rightarrow a = d$

$\therefore\ A = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$

4. (5 points) Compute the following three matrices:

$$\begin{bmatrix} 1 & 0 & 0 \\ \alpha & 1 & 0 \\ \beta & 0 & 1 \end{bmatrix}^n, \quad \begin{bmatrix} 1 & 0 & 0 \\ \alpha & 1 & 0 \\ \beta & 0 & 1 \end{bmatrix}^{-1}, \quad \begin{bmatrix} 1 & 0 & 0 \\ \alpha & 1 & 0 \\ 0 & \beta & 1 \end{bmatrix}^{-1}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ n\alpha & 1 & 0 \\ n\beta & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ -\alpha & 1 & 0 \\ -\beta & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ -\alpha & 1 & 0 \\ \alpha\beta & -\beta & 1 \end{bmatrix}$$