Engineering Mathematics I

Midterm Exam, October 19, 2016

Problem	Score
1	
2	
3	
4	
5	
6	
7	
Total	

Name: ______________________

ID No: ______________________

Dept: ______________________

E-mail: ______________________

1. (15 points) A young man with no initial capital invests k dollars per year at an annual rate of return r. Assume that investments are made continuously and that the return is compounded continuously.

   (a) (10 points) Determine the sum $S(t)$ accumulated at any time t.

   (b) (5 points) If $r = 7.5\%$, determine k so that \$1 million will be available for retirement in 40 years.

2. (20 points) Solve the following initial value problem (without using Laplace transforms):

$$\begin{aligned} y_1' &= y_2 + 2e^t, \quad y_1(0) = 1, \\ y_2' &= -y_1 + 2y_2 + 3e^t, \quad y_2(0) = 1. \end{aligned}$$

3. (10 points) Solve the following initial value problem by the power series method. Find the recurrence formula and find the first five nonzero terms in the series.

$$y'' - 2xy' - 2y = 0, \quad y(0) = 1, \; y'(0) = 0.$$

4. (20 points) Consider the following integral equation:

$$y(t) + 2\int_0^t \cos(t-\tau)y(\tau)d\tau = e^{-t}. \tag{1}$$

(a) (5 points) Solve Equation (1) using the Laplace transformation.

(b) (10 points) By differentiating Equation (1) twice, show that $y(t)$ satisfies the following initial value problem:

$$y'' + 2y' + y = 2e^{-t}, \quad y(0) = 1,\ y'(0) = -3. \tag{2}$$

(c) (5 points) Solve Equation (2) and verify that the solution is the same as in Equation (1).

5. (15 points) Using Laplace transforms, solve the following system of differential equations

$$\begin{aligned} y_1' + \ y_2' + y_1 + y_2 &= 1, \qquad y_1(0) = 0, \\ y_1' + 2y_2' + \qquad y_2 &= 0, \qquad y_2(0) = 1. \end{aligned}$$

6. (10 points) Find the following transformations:

   (a) (3 points) $\mathcal{L}^{-1}\left[\frac{d^n}{ds^n}\frac{1}{s^2+\omega^2}\right]$, for $n = 1, 2, 3, \cdots$

   (b) (2 points) $\mathcal{L}^{-1}\left[\frac{d^n}{ds^n}\frac{s}{s^2+\omega^2}\right]$, for $n = 1, 2, 3, \cdots$

   (c) (5 points) $\mathcal{L}\left[te^{at}\sin\omega t\right]$

7. (10 points) Using Laplace transforms, solve the following initial value problem:

$$y'' + y = \delta(t-\pi)e^{2t}, \quad y(0) = 0,\ y'(0) = 1.$$