Quiz #1 (EngMath I) [Monday, Sept. 12, 2016]

Name: _____ ID No: _____

1. (7 points) Solve the following differential equation:

$$\left(\frac{2x}{y} - \frac{y}{x^2+y^2}\right)dx + \left(\frac{x}{x^2+y^2} - \frac{x^2}{y^2}\right)dy = 0$$

$$\frac{\partial M}{\partial y} = -\frac{2x}{y^2} - \frac{x^2+y^2-2y^2}{(x^2+y^2)^2} = -\frac{2x}{y^2} - \frac{x^2-y^2}{(x^2+y^2)^2} \quad \text{+1}$$

$$\frac{\partial N}{\partial x} = \frac{x^2+y^2-2x^2}{(x^2+y^2)^2} - \frac{2x}{y^2} = \frac{y^2-x^2}{(x^2+y^2)^2} - \frac{2x}{y^2} \quad \text{+1}$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} : \text{exact}$$

$$u(x,y) = \int M(x,y)\,dx + k(y)$$

$$= \frac{x^2}{y} - \arctan\left(\frac{x}{y}\right) + k(y) \quad \text{+2}$$

$$\frac{\partial u}{\partial y} = -\frac{x^2}{y^2} - \frac{-\frac{x}{y^2}}{\left(\frac{x}{y}\right)^2+1} + k'(y)$$

$$= N(x,y)$$

$$\therefore k'(y) = 0 \,,\; k(y) = c^* \quad \text{+1}$$

$$\therefore u(x,y) = \frac{x^2}{y} - \arctan\left(\frac{x}{y}\right) = c \quad \text{+1}$$

Check: $\frac{\partial u}{\partial x} = \frac{2x}{y^2} - \frac{\frac{1}{y}}{\left(\frac{x}{y}\right)^2+1} = \frac{2x}{y^2} - \frac{y}{x^2+y^2} = M(x,y)$

$$\frac{\partial u}{\partial y} = -\frac{x^2}{y^2} - \frac{-\frac{x}{y^2}}{\left(\frac{x}{y}\right)^2+1} = \frac{x}{x^2+y^2} - \frac{x^2}{y^2} = N(x,y) \quad \text{+1}$$

2. (8 points) Solve the following initial value problem:

$$\frac{dy}{dx} + 2xy = e^{-x^2-2x}, \quad y(0) = 3.$$

$y' + 2xy = e^{-x^2-2x}$

$p(x) = 2x$ ---------- (+1)

$h(x) = \int p(x)\,dx = \int 2x\,dx = x^2$ ---------- (+1)

$y = e^{-x^2}\left[\int e^{x^2} \cdot e^{-x^2-2x}\,dx + C\right]$ ---------- (+2)

$= e^{-x^2}\left[\int e^{-2x}\,dx + C\right]$

$= C \cdot e^{-x^2} + e^{-x^2}\left(-\frac{1}{2}e^{-2x}\right)$ } (+2)

$= C \cdot e^{-x^2} - \frac{1}{2}e^{-x^2-2x}$

$y(0) = C - \frac{1}{2} = 3$

$C = \frac{7}{2}$

$\therefore y(x) = \frac{7}{2}e^{-x^2} - \frac{1}{2}e^{-x^2-2x}$ ---------- (+1)

Check:

$y' + 2xy = \frac{7}{2} \cdot (-2x) \cdot e^{-x^2} - \frac{1}{2}(-2x-2) \cdot e^{-x^2-2x}$

$+ 2x \cdot \left(\frac{7}{2}e^{-x^2} - \frac{1}{2}e^{-x^2-2x}\right)$

$= e^{-x^2-2x}$ (+1)

$y(0) = \frac{7}{2} - \frac{1}{2}$

$= 3$