Quiz #2 (CSE4190.410)

September 27, 2010 (Monday)

Name: ____________ **Dept:** ____________ **ID No:** ____________

1. (10 points) Consider a tetrahedron with four vertices $A(0,0,0)$, $B(1,0,0)$, $C(0,1,0)$, and $D(0,0,1)$. Apply a perspective projection to this tetrahedron onto the viewplane $3x-4y+5z=0$ from the viewpoint $(1,-2,3)$ and compute the four projected points A', B', C', and D'.

$$\hat{n} = (3,-4,5,0), \quad \hat{v} = (1,-2,3,1)$$

$$\hat{x}' = \hat{n} \times (\hat{x} \times \hat{v}) = \langle \hat{n}, \hat{v} \rangle \hat{x} - \hat{v} \langle \hat{n}, \hat{x} \rangle$$

$$= 26\hat{x} - \begin{bmatrix} 3 & -4 & 5 & 0 \\ -6 & 8 & -10 & 0 \\ 9 & -12 & 15 & 0 \\ 3 & -4 & 5 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 23 & 4 & -5 & 0 \\ 6 & 18 & 10 & 0 \\ -9 & 12 & 11 & 0 \\ -3 & 4 & -5 & 26 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \hat{A}' & \hat{B}' & \hat{C}' & \hat{D}' \end{bmatrix} = \begin{bmatrix} 0 & 23 & 4 & -5 \\ 0 & 6 & 18 & 10 \\ 0 & -9 & 12 & 11 \\ 26 & 23 & 30 & 21 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 2/15 & -5/21 \\ 0 & 6/23 & 3/5 & 10/21 \\ 0 & -9/23 & 2/5 & 11/21 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

2. (5 points) Consider two parallel planes:

$$\Pi_1: \quad 3x - 4y + 5z + 6 = 0,$$
$$\Pi_2: \quad 3x - 4y + 5z - 6 = 0.$$

What is the affine transformation from R^3 to R^1 that sends Π_1 to 0 and Π_2 to 1?

$$\begin{bmatrix} 3 & -4 & 5 & 6 \\ 0 & 0 & 0 & 12 \end{bmatrix}$$

3. (5 points) Write down an algebraic expression for testing whether the following four planes share a common intersection point:

$$\Pi_i: \quad a_i x + b_i y + c_i z + d_i = 0, \quad (i = 1, 2, 3, 4).$$

$$\begin{vmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{vmatrix} = 0$$

Quiz #2 (CSE4190.410)

September 28, 2011 (Wednesday)

Name: _______________ **Dept:** _______________ **ID No:** _______________

1. (5 points) What is the perspective projection of a point $\mathbf{p} = (3, 5, 7)$ from the view point $\mathbf{v} = (1, 2, 3)$ onto the line $x + y + z + 1 = 0$?

$$\hat{\mathbb{p}} = (3, 5, 7, 1)$$

$$\hat{\mathbb{v}} = (1, 2, 3, 1)$$

$$\hat{\mathbb{n}} = (1, 1, 1, 1)$$

$$\begin{aligned} \hat{\mathbb{n}} \times (\hat{\mathbb{p}} \times \hat{\mathbb{v}}) &= \langle \hat{\mathbb{n}}, \hat{\mathbb{v}} \rangle \hat{\mathbb{p}} - \langle \hat{\mathbb{n}}, \hat{\mathbb{p}} \rangle \hat{\mathbb{v}} \\ &= 7\hat{\mathbb{p}} - 16\hat{\mathbb{v}} \\ &= (5, 3, 1, -9) \\ &= \left(-\frac{5}{9}, -\frac{3}{9}, -\frac{1}{9}, 1\right) \end{aligned}$$

$$\therefore \left(-\frac{5}{9}, -\frac{3}{9}, -\frac{1}{9}\right)$$

2. (7 points) Consider two parallel planes:

$$\Pi_1 : \quad ax + by + cz + d_1 = 0,$$
$$\Pi_2 : \quad ax + by + cz + d_2 = 0.$$

What is the affine transformation from R^3 to R^1 that sends Π_1 to -1 and Π_2 to 1?

$$\begin{bmatrix} 2a & 2b & 2c & d_1+d_2 \\ 0 & 0 & 0 & d_1-d_2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2ax+2by+2cz+d_1+d_2 \\ d_1-d_2 \end{bmatrix}$$

For $(x,y,z) \in \Pi_1$, we have

$$2(ax+by+cz) = -2d_1$$

$$\therefore 2ax+2by+2cz+d_1+d_2 = d_2-d_1$$

For $(x,y,z) \in \Pi_2$, we have

$$2(ax+by+cz) = -2d_2$$

$$\therefore 2ax+2by+2cz+d_1+d_2 = d_1-d_2$$

Hence, the above affine transformation sends Π_1 and Π_2 to -1 and 1, respectively.

3. (8 points) Using the wedge-product operation discussed in class, answer the following questions. What is the plane that is determined by three points $(1, 2, 3)$, $(3, 5, 7)$, and $(2, 3, 5)$? What is its intersection with other planes $x + y + z + 1 = 0$ and $x - y - z + 1 = 0$?

$$(1,2,3,1) \wedge (3,5,7,1) \wedge (2,3,5,1)$$

$$= (2,0,-1,1)$$

$$\therefore 2x - z + 1 = 0$$

$$(2,0,-1,1) \wedge (1,1,1,1) \wedge (1,-1,-1,1)$$

$$= (2,-2,2,-2)$$

$$= (-1,1,-1,1)$$

$$\therefore (-1,1,-1)$$

Quiz #2 (CSE4190.410)

September 24, 2012 (Monday)

1. (10 points) Consider two parallel planes:

$$\Pi_1 : \quad ax + by + cz + d_1 = 0,$$
$$\Pi_2 : \quad ax + by + cz + d_2 = 0.$$

   (a) (4 points) What is the affine transformation from R^3 to R^1 that sends Π_1 to d_1 and Π_2 to d_2?

   (b) (2 points) What is the 1D translation that sends d_1 to 0?

   (c) (2 points) What is the 1D uniform scaling by a factor $\frac{1}{d_2-d_1}$?

   (d) (2 point) What is the composite affine transformation of the above three?

(a) $\begin{bmatrix} -a & -b & -c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ or

$\begin{bmatrix} a & b & c & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -d_1 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 0 & d_2-d_1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 \\ 0 & d_2-d_1 \end{bmatrix}\begin{bmatrix} 1 & -d_1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a & b & c & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} a & b & c & d_1 \\ 0 & 0 & 0 & d_1-d_2 \end{bmatrix}$

Quiz #2 (CSE4190.410)

September 25, 2013 (Wednesday)

1. (10 points) Consider three parallel planes:

$$\Pi_i: \quad ax + by + cz + d_i = 0, \qquad (i = 0, 1, 2).$$

(a) (1 points) What is the affine transformation from R^3 to R^1 that sends Π_0 to d_0, Π_1 to d_1, and Π_2 to d_2?

(b) (1 points) What is the 1D translation that sends d_0 to 0, d_1 to $\bar{d}_1 = d_1 - d_0$, and d_2 to $\bar{d}_2 = d_2 - d_0$?

(c) (7 points) What is the 1D perspective transformation that sends 0 to 0, $\bar{d}_1$ to 1, and $\bar{d}_2$ to 2?

(d) (1 point) What is the composite perspective transformation from R^3 to R^1 that sends Π_0 to 0, Π_1 to 1, and Π_2 to 2?

(a) $\begin{bmatrix} a & b & c & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -d_0 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} A & B \\ C & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} B \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $B = 0$ +1

$\begin{bmatrix} A & 0 \\ C & 1 \end{bmatrix}\begin{bmatrix} \bar{d}_1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow \frac{A\bar{d}_1}{C\bar{d}_1 + 1} = 1 \quad \therefore A = C + \frac{1}{\bar{d}_1}$

$\begin{bmatrix} A & 0 \\ C & 1 \end{bmatrix}\begin{bmatrix} \bar{d}_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \Rightarrow \frac{A\bar{d}_2}{C\bar{d}_2 + 1} = 2 \quad \therefore A = 2C + \frac{2}{\bar{d}_2}$ +2

$C + \frac{1}{\bar{d}_1} = 2C + \frac{2}{\bar{d}_2} \Rightarrow C = \frac{1}{\bar{d}_1} - \frac{2}{\bar{d}_2} = \frac{\bar{d}_2 - 2\bar{d}_1}{\bar{d}_1\bar{d}_2}$ +2

$A = \frac{2}{\bar{d}_1} - \frac{2}{\bar{d}_2} = \frac{2(\bar{d}_2 - \bar{d}_1)}{\bar{d}_1\bar{d}_2}$ +2

$\therefore \begin{bmatrix} 2(\bar{d}_2 - \bar{d}_1) & 0 \\ \bar{d}_2 - 2\bar{d}_1 & \bar{d}_1\bar{d}_2 \end{bmatrix}$

(d) $\begin{bmatrix} 2(\bar{d}_2 - \bar{d}_1) & 0 \\ \bar{d}_2 - 2\bar{d}_1 & \bar{d}_1\bar{d}_2 \end{bmatrix}\begin{bmatrix} 1 & -d_0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a & b & c & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

Quiz #2 (CSE4190.410)

October 5, 2015 (Monday)

Name: ______________ **Dept:** ______________ **ID No:** ______________

1. (10 points) A transformation from R^1 to R^1 sends 1 to 1; 2 to 2; and 3 to 7.
   (a) (5 points) What is the matrix representation of this transformation?
   (b) (3 points) This transformation sends t to $f(t)$. What is the representation of $f(t)$ as a linear rational function of t?
   (c) (2 points) Which value of t goes to infinity under this transformation?

(a) $\begin{bmatrix} A & B \\ C & 1 \end{bmatrix} \begin{bmatrix} t_i \\ 1 \end{bmatrix} = \begin{bmatrix} f_i \\ 1 \end{bmatrix}$, $i = 1, 2, 3$

$$A t_i + B = f_i (C t_i + 1)$$

$$\begin{cases} A + B - C = 1 \\ 2A + B - 4C = 2 \\ 3A + B - 21C = 7 \end{cases} \begin{matrix} \Rightarrow A - 3C = 1 \\ \Rightarrow A - 17C = 5 \end{matrix}$$

$$\therefore A = \frac{1}{7}, \quad C = -\frac{2}{7}, \quad B = \frac{4}{7}$$

$$\begin{bmatrix} 1 & 4 \\ -2 & 7 \end{bmatrix}$$

(b) $f(t) = \dfrac{t + 4}{-2t + 7}$

(c) $t = \dfrac{7}{2}$

Quiz #2 (CSE4190.410)

October 1, 2010 (Wednesday)

Name: ______________ Dept: ______________ ID No: ______________

1. (5 points) Write down an algebraic expression for testing whether a given direction vector $\mathbf{d} = (d_1, d_2, d_3)$ is parallel to the plane determined by three non-colinear points $\mathbf{p}_i = (x_i, y_i, z_i)$, $i = 1, 2, 3$.

$$\begin{vmatrix} d_1 & d_2 & d_3 & 0 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} = 0$$

2. (5 points) Consider three parallel planes:

$$\begin{aligned} \Pi_0 &: \quad 3x - 4y + 5z = 0, \\ \Pi_1 &: \quad 3x - 4y + 5z + 1 = 0, \\ \Pi_2 &: \quad 3x - 4y + 5z + 2 = 0. \end{aligned}$$

What is the transformation from R^3 to R^1 that sends Π_0 to 0, Π_1 to 1, and Π_2 to 3?

$$\begin{bmatrix} A & 0 \\ B & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow A = B + 1$$

$$\begin{bmatrix} A & 0 \\ B & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \Rightarrow 2A = 6B + 3$$

$$\therefore B = -\frac{1}{4}, \quad A = \frac{3}{4}$$

$$\begin{bmatrix} 3 & 0 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} -3 & 4 & -5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & 12 & -15 & 0 \\ 3 & -4 & 5 & 4 \end{bmatrix}$$