Quiz #1 (Comp 4190.504)

March 10, 2010 (Wednesday)

Name: _____   Dept: _____   ID No: _____

1. (5 points) Using the wedge operation discussed in class, answer the following questions. What is the line that is determined by two points $(3, 4)$ and $(5, 6)$? What is its intersection with another line $x + y + 1 = 0$?

$$\hat{x}_1 = (3, 4, 1)$$

$$\hat{x}_2 = (5, 6, 1)$$

$$\hat{x}_1 \times \hat{x}_2 = (-2, 2, -2) = (1, -1, 1)$$

$$\hat{m}_1 = (1, -1, 1)$$

$$\hat{m}_2 = (1, 1, 1)$$

$$\hat{m}_1 \times \hat{m}_2 = (-2, 0, 2) = (-1, 0, 1)$$

$$\therefore (-1, 0) \text{ is the intersection point.}$$

2. (5 points) Write down an algebraic expression for testing whether the following four planes share a common intersection point:

$$\Pi_i : \quad a_i x + b_i y + c_i z + d_i = 0, \quad (i = 1, 2, 3, 4).$$

$$\hat{\mathbb{m}}_2 \wedge \hat{\mathbb{m}}_3 \wedge \hat{\mathbb{m}}_4$$

$$= \left(\begin{vmatrix} b_2 & c_2 & d_2 \\ b_3 & c_3 & d_3 \\ b_4 & c_4 & d_4 \end{vmatrix}, \; - \begin{vmatrix} a_2 & c_2 & d_2 \\ a_3 & c_3 & d_3 \\ a_4 & c_4 & d_4 \end{vmatrix}, \; \begin{vmatrix} a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \\ a_4 & b_4 & d_4 \end{vmatrix}, \; - \begin{vmatrix} a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \\ a_4 & b_4 & d_4 \end{vmatrix} \right)$$

The four planes share a common intersection point

$$\iff \quad a_1 \begin{vmatrix} b_2 & c_2 & d_2 \\ b_3 & c_3 & d_3 \\ b_4 & c_4 & d_4 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 & d_2 \\ a_3 & c_3 & d_3 \\ a_4 & c_4 & d_4 \end{vmatrix}$$

$$+ \, c_1 \begin{vmatrix} a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \\ a_4 & b_4 & d_4 \end{vmatrix} - d_1 \begin{vmatrix} a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \\ a_4 & b_4 & d_4 \end{vmatrix} = 0$$

$$\iff \quad \begin{vmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{vmatrix} = 0$$