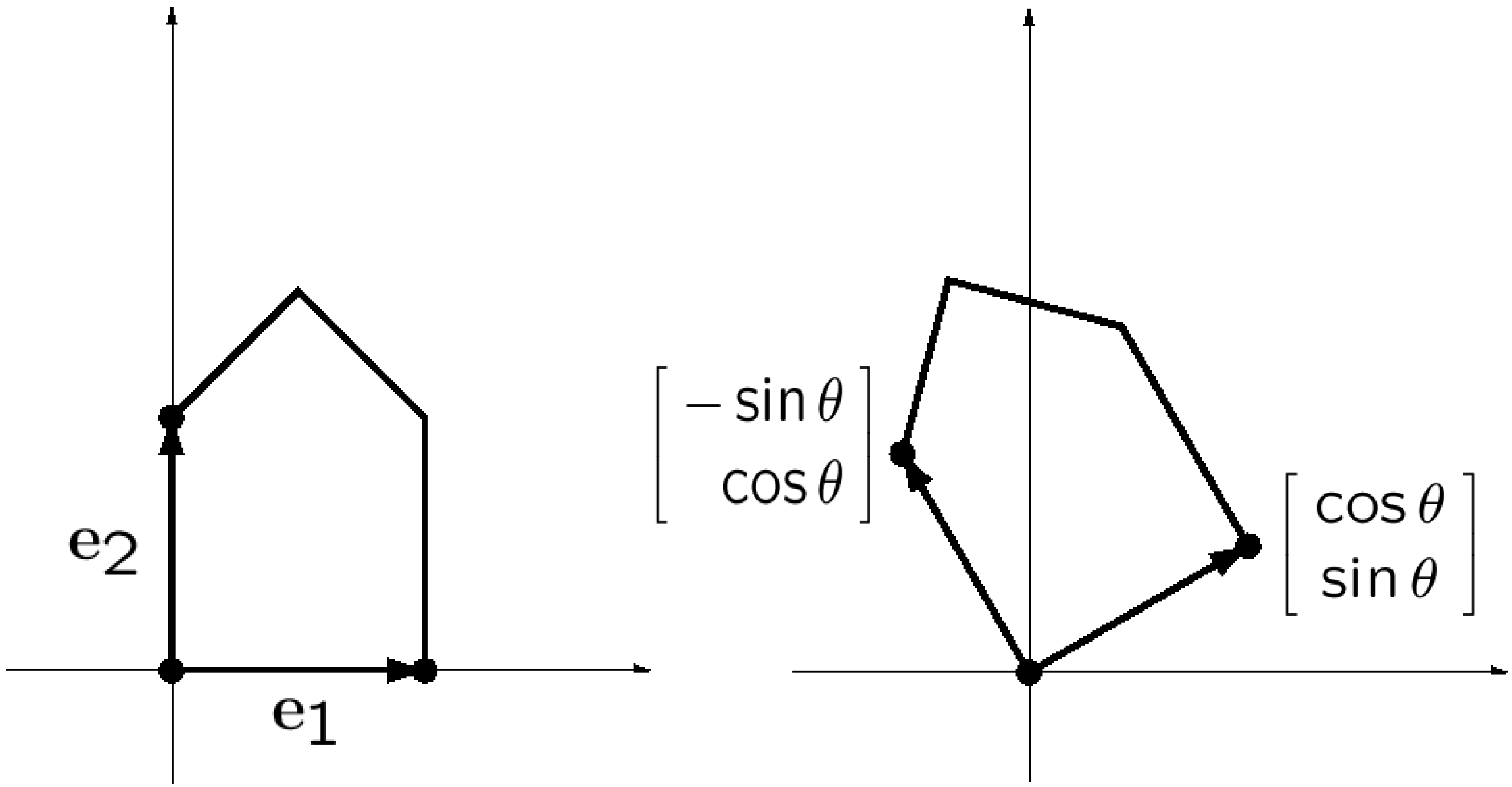
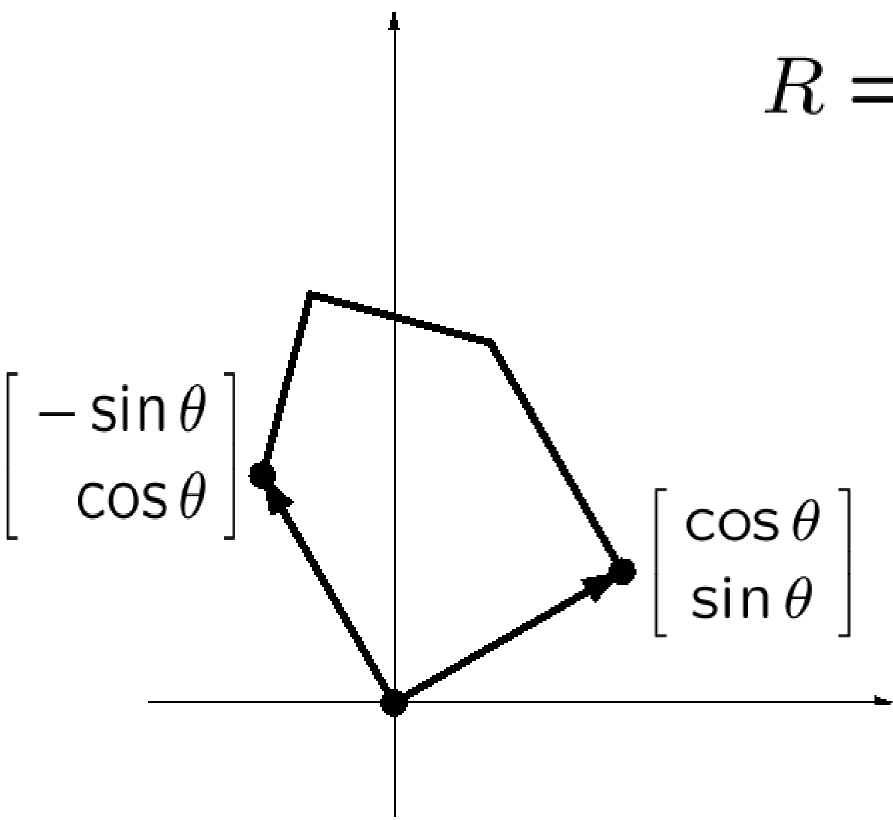
행렬과 2차원 변환

서울대학교 컴퓨터공학부
김명수
http://cse.snu.ac.kr/mskim
http://3map.snu.ac.kr

2차원 회전


e_2
e_1
$\begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$
$\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$

2차원 회전

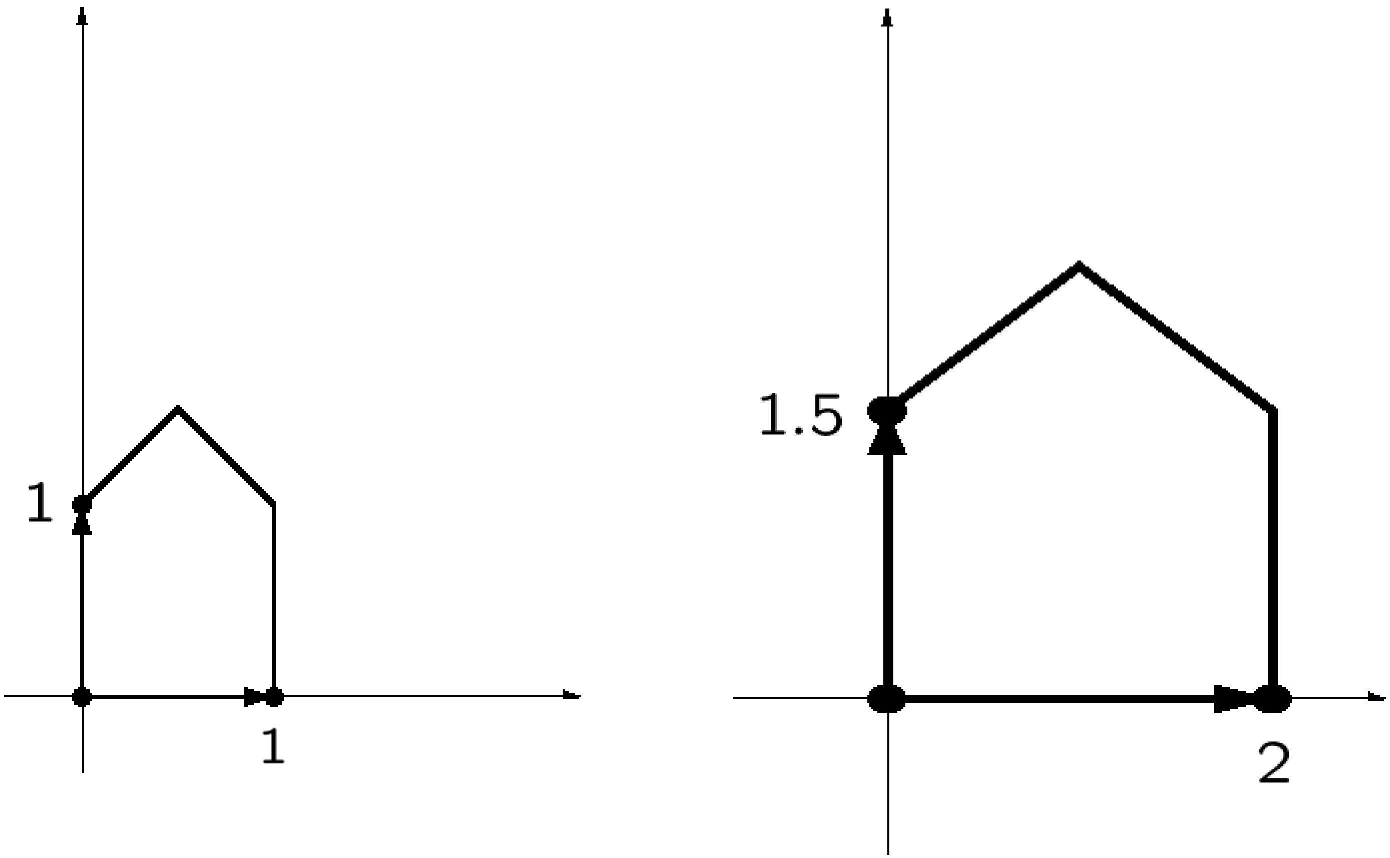


$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

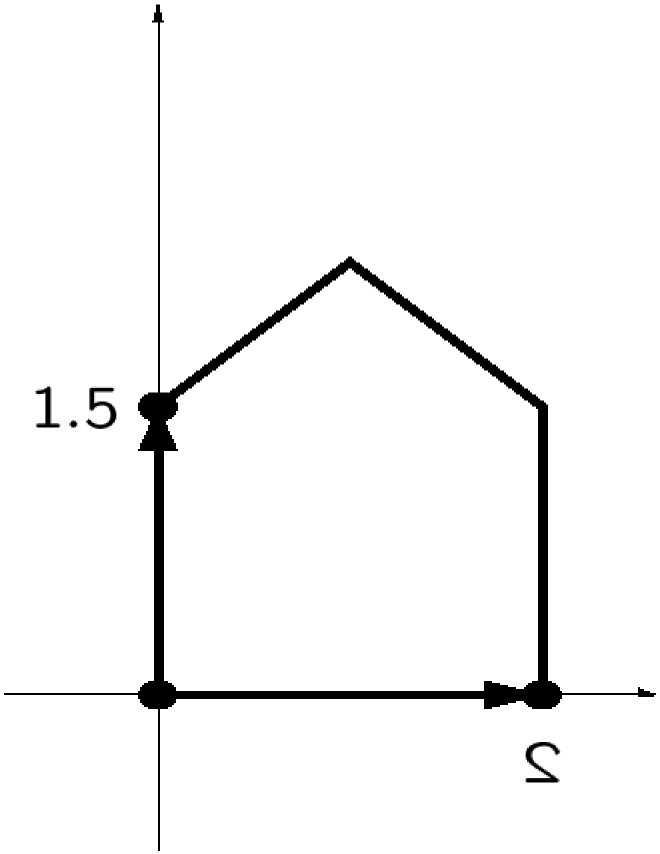
$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$$

2차원 축소확대



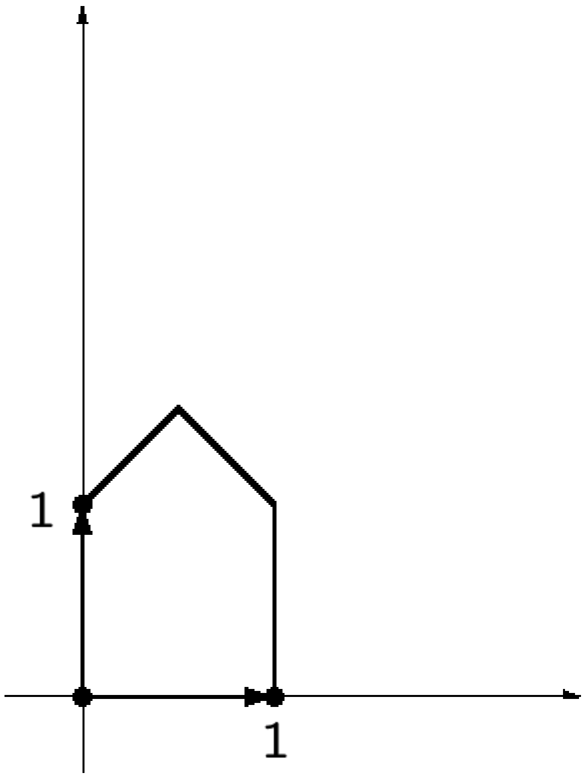
2차원 축소확대



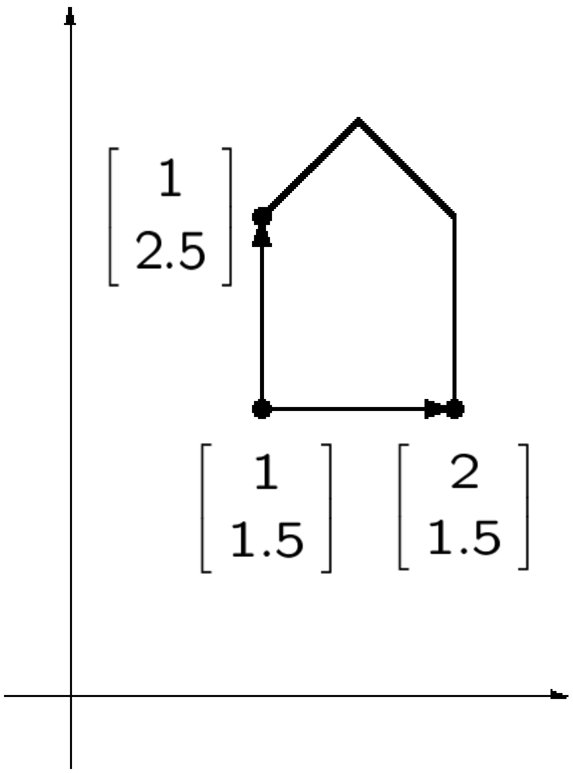
$$S = \begin{bmatrix} 2 & 0 \\ 0 & 1.5 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 1.5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x \\ 1.5y \end{bmatrix}$$

2차원 평행이동

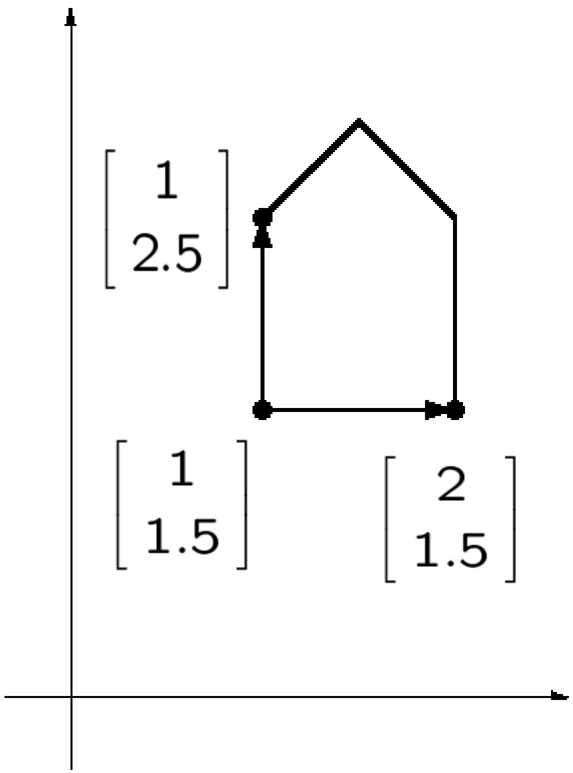


1
1



$\begin{bmatrix} 1 \\ 2.5 \end{bmatrix}$
$\begin{bmatrix} 1 \\ 1.5 \end{bmatrix}$
$\begin{bmatrix} 2 \\ 1.5 \end{bmatrix}$

2차원 평행이동



$$\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1.0 \\ 1.5 \end{bmatrix} = \begin{bmatrix} x + 1.0 \\ y + 1.5 \end{bmatrix}$$

2차원 변환

$$\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = \begin{bmatrix} t_x \\ t_y \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix}$$

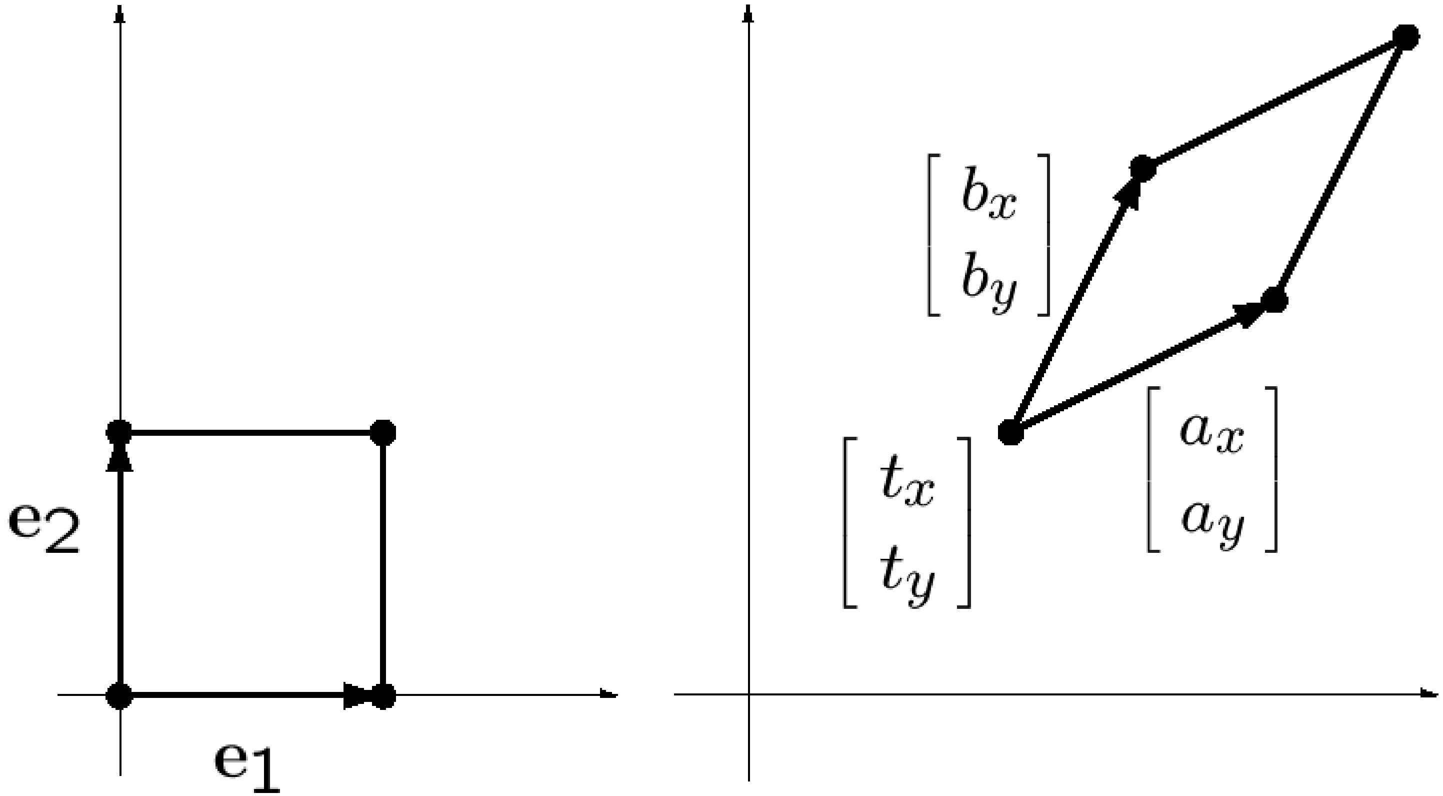
Homogeneous 좌표계

$$\begin{bmatrix} \hat{x} \\ \hat{y} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \hat{x} \\ \hat{y} \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \hat{x} \\ \hat{y} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2차원 변환



e_2
e_1
$\begin{bmatrix} b_x \\ b_y \end{bmatrix}$
$\begin{bmatrix} t_x \\ t_y \end{bmatrix}$
$\begin{bmatrix} a_x \\ a_y \end{bmatrix}$

2차원 변환

$$\begin{bmatrix} \hat{x} \\ \hat{y} \\ 1 \end{bmatrix} = \begin{bmatrix} a_x & b_x & t_x \\ a_y & b_y & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = \begin{bmatrix} a_x & b_x \\ a_y & b_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

2차원 변환

$$\begin{bmatrix} t_x \\ t_y \\ 1 \end{bmatrix} = \begin{bmatrix} a_x & b_x & t_x \\ a_y & b_y & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

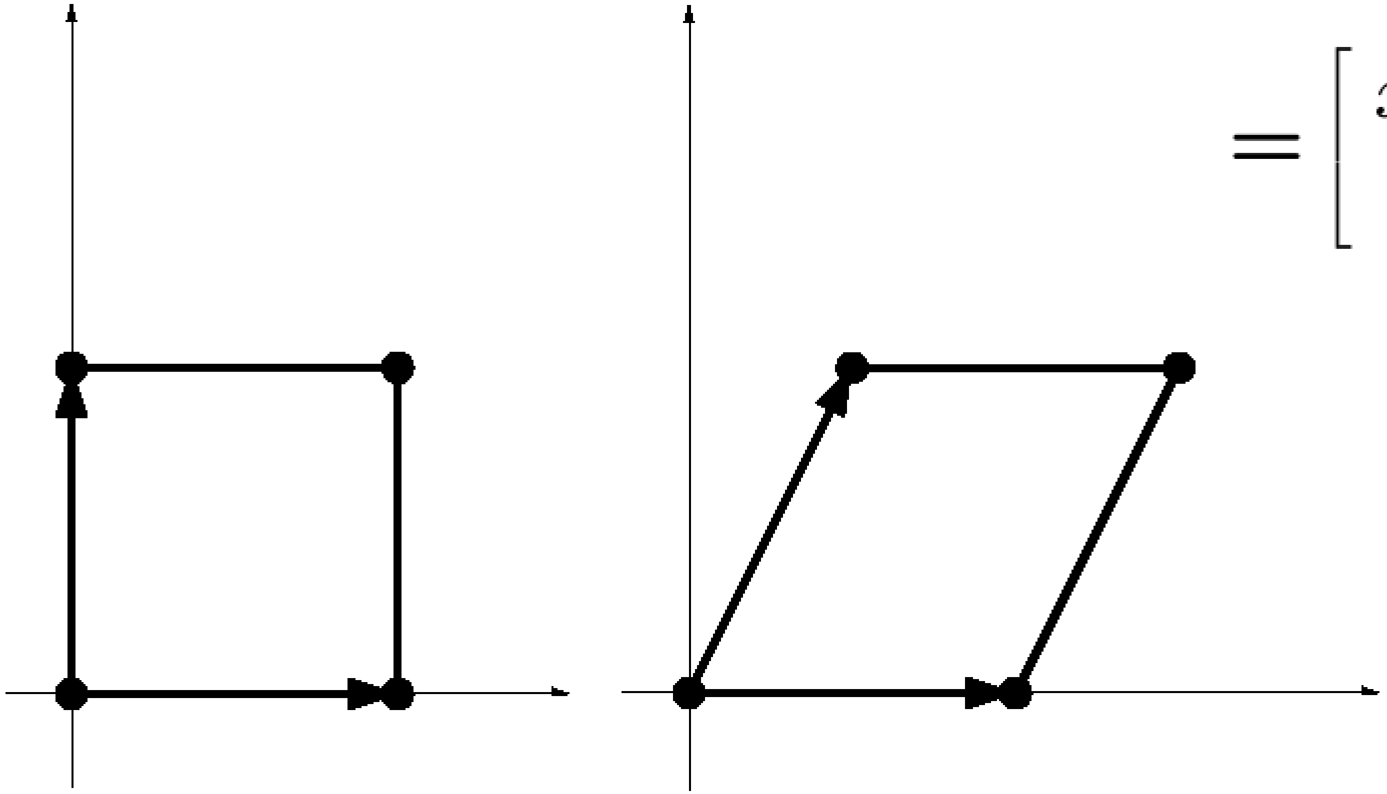
$$\begin{bmatrix} a_x \\ a_y \\ 0 \end{bmatrix} = \begin{bmatrix} a_x & b_x & t_x \\ a_y & b_y & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} b_x \\ b_y \\ 0 \end{bmatrix} = \begin{bmatrix} a_x & b_x & t_x \\ a_y & b_y & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

2차원 x-shearing

$$\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = \begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

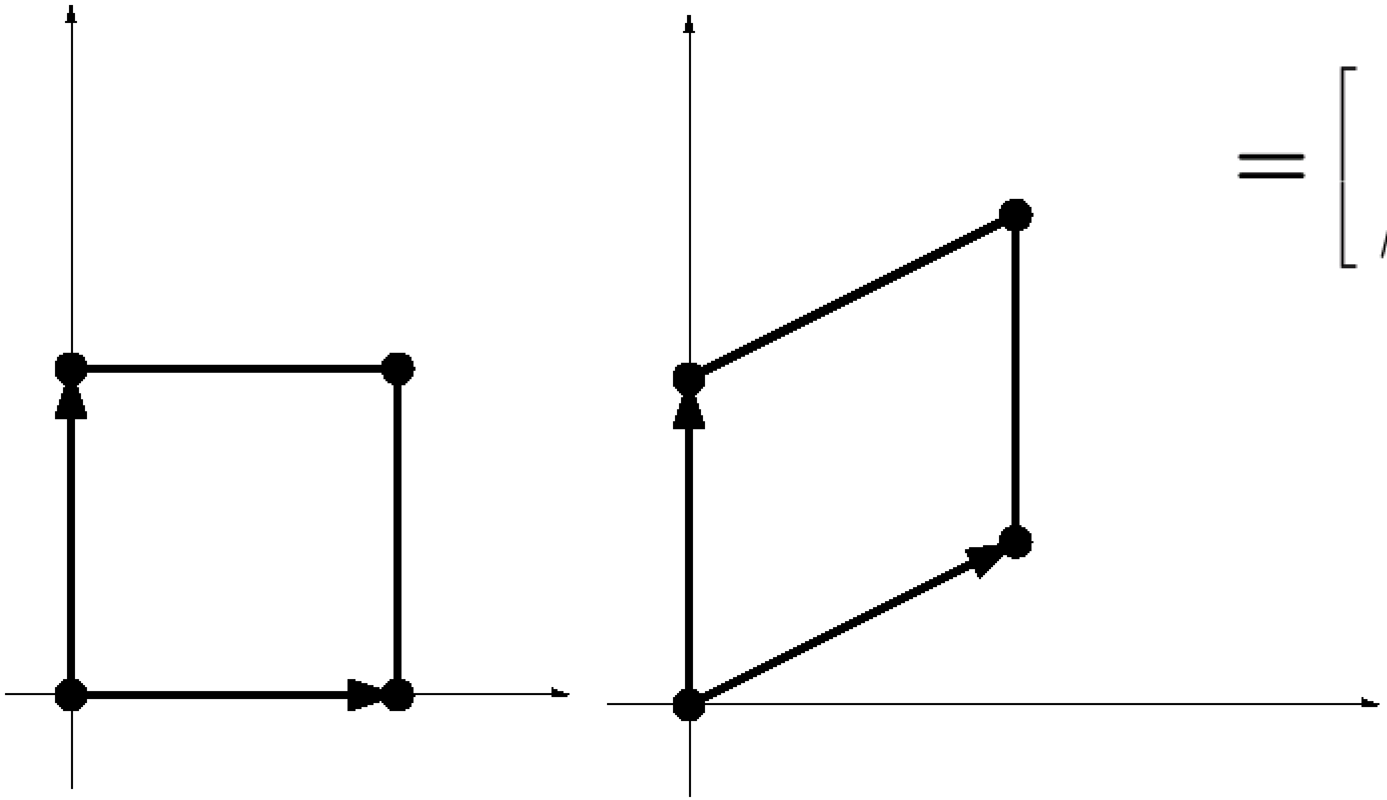
$$= \begin{bmatrix} x + \alpha y \\ y \end{bmatrix}$$



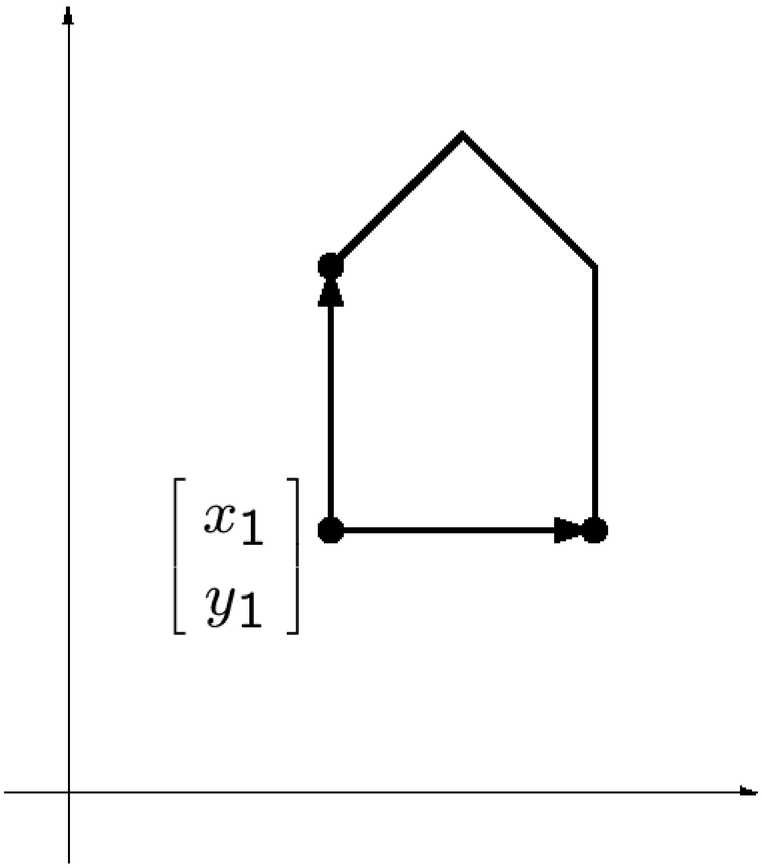
2차원 y-shearing

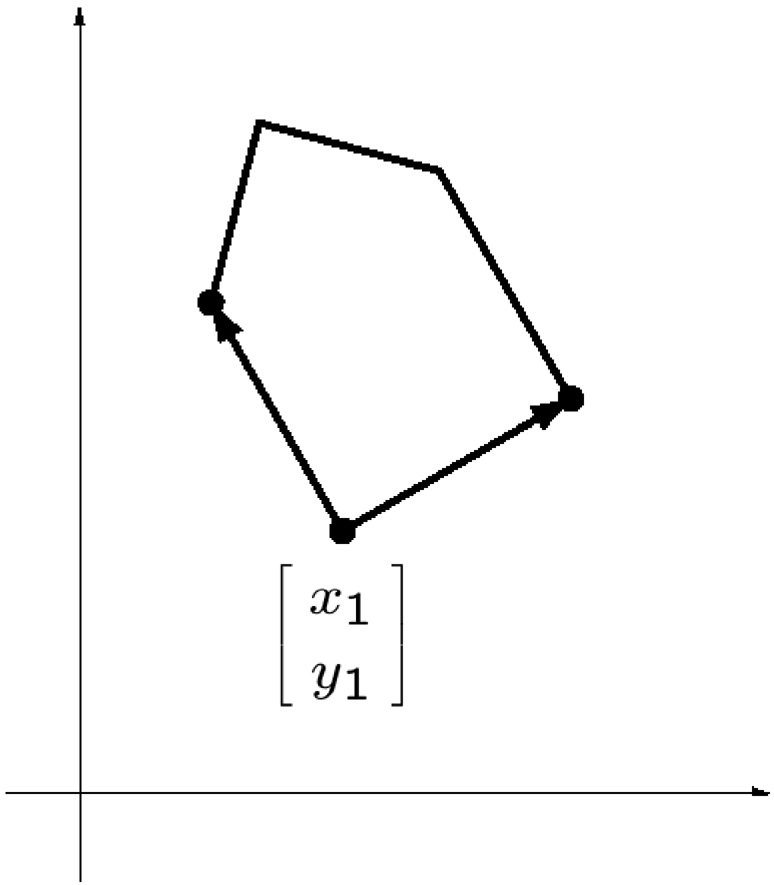
$$\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \beta & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} x \\ \beta x + y \end{bmatrix}$$

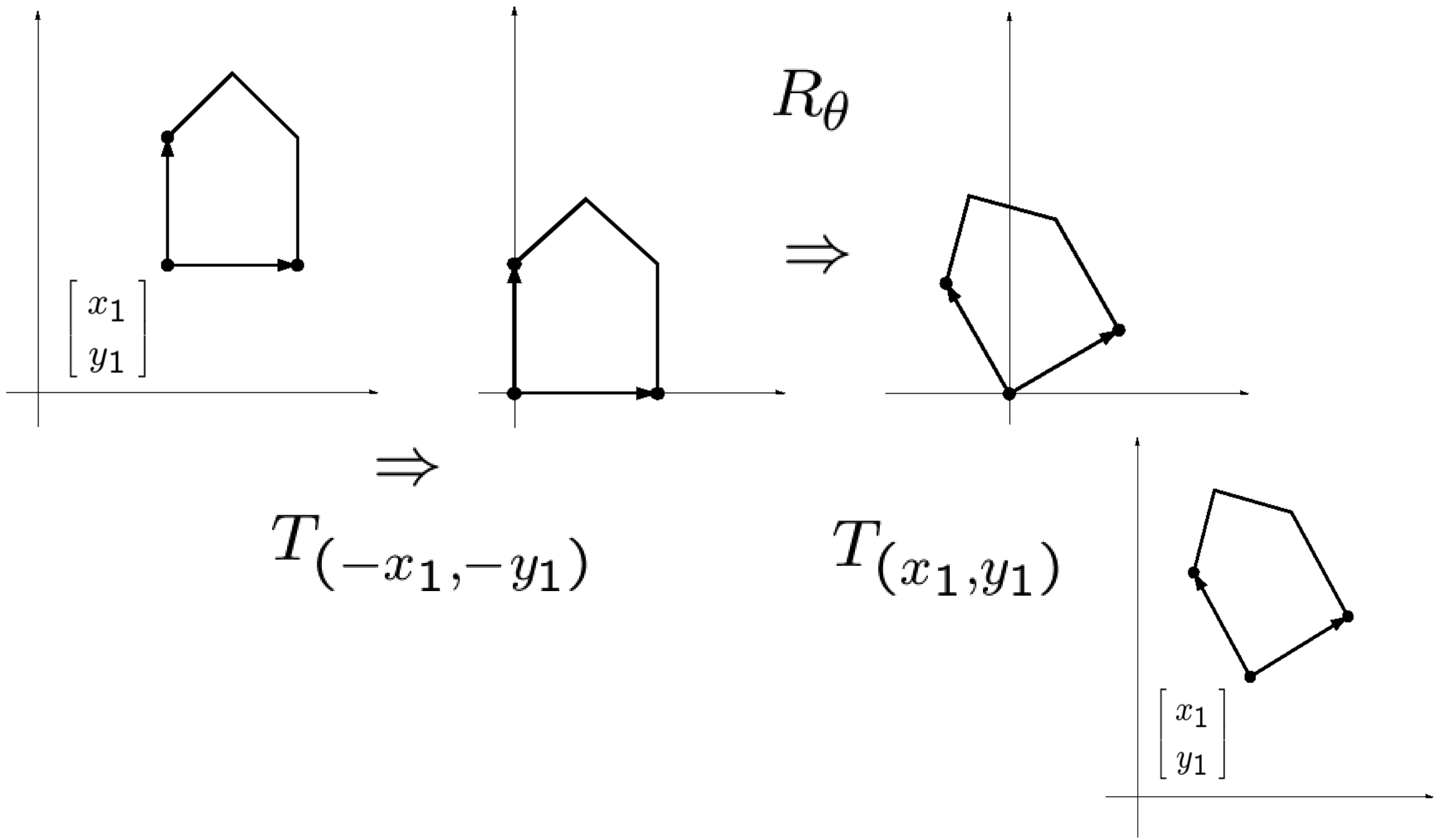


2차원 일반회전


$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$


$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$

2차원 일반회전



$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$
$\Rightarrow$
$T_{(-x_1,-y_1)}$
R_θ
$\Rightarrow$
$T_{(x_1,y_1)}$
$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$

2차원 일반회전

$$\begin{bmatrix} 1 & 0 & x_1 \\ 0 & 1 & y_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_1 \\ 0 & 1 & -y_1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & x_1 \\ 0 & 1 & y_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & -x_1\cos\theta + y_1\sin\theta \\ \sin\theta & \cos\theta & -x_1\sin\theta - y_1\cos\theta \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & x_1(1-\cos\theta) + y_1\sin\theta \\ \sin\theta & \cos\theta & y_1(1-\cos\theta) - x_1\sin\theta \\ 0 & 0 & 1 \end{bmatrix}$$

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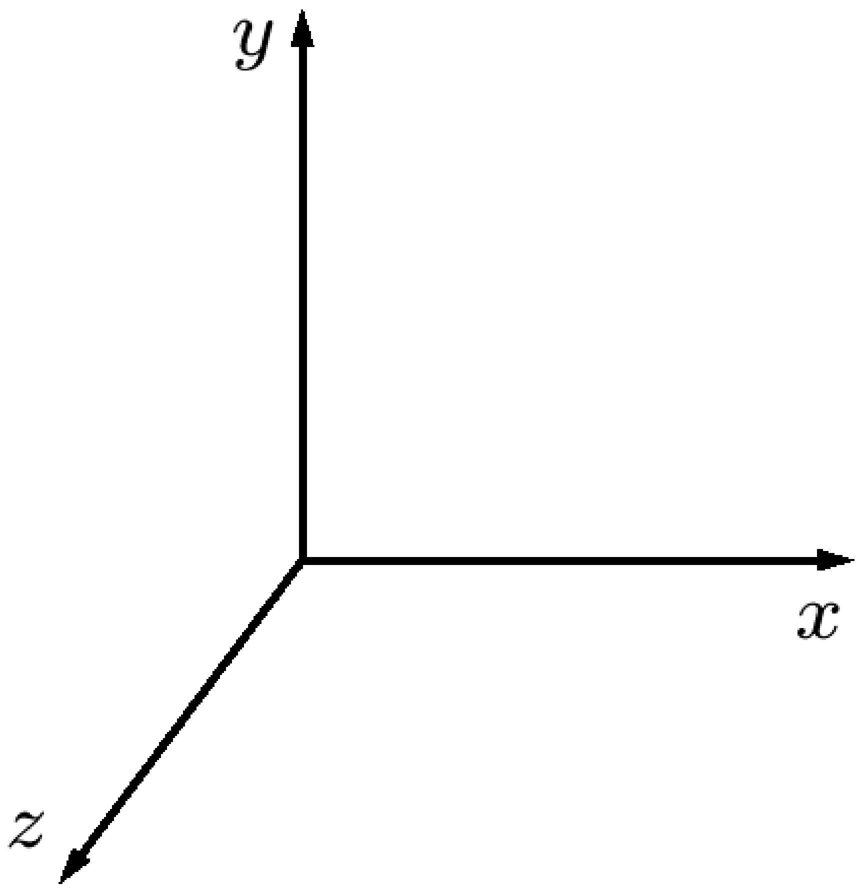
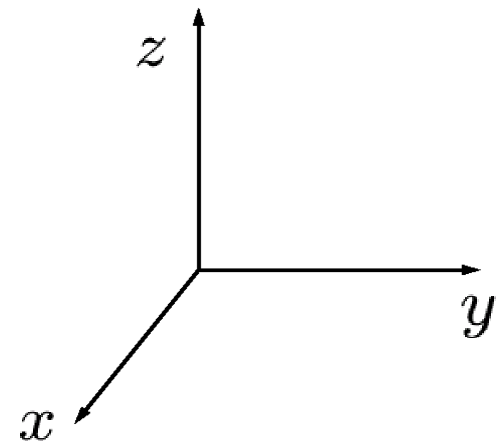
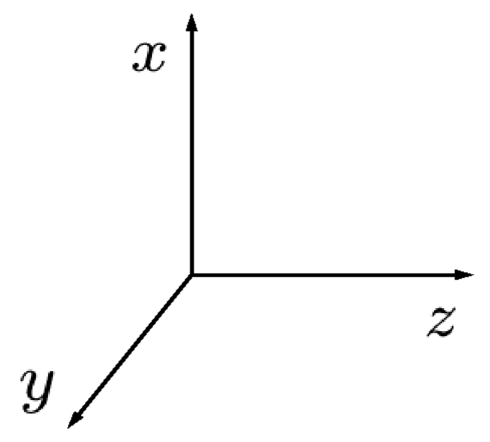
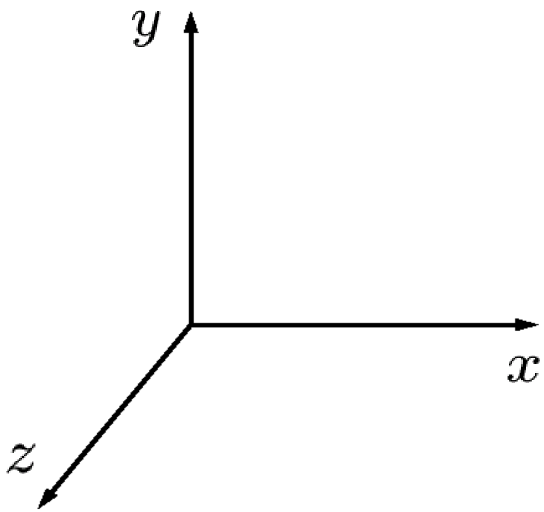
오른손 좌표계

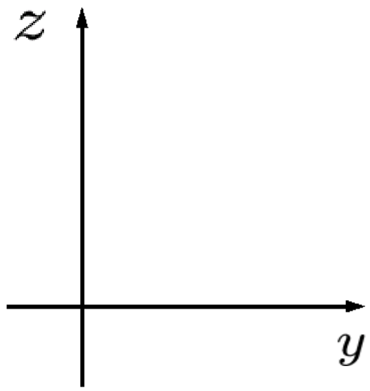


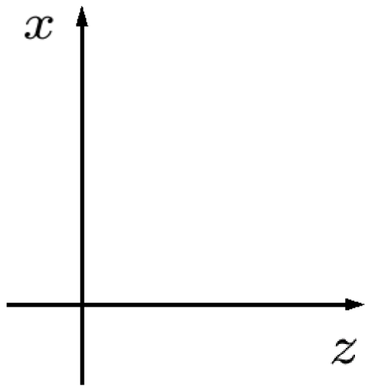




오른손 좌표계

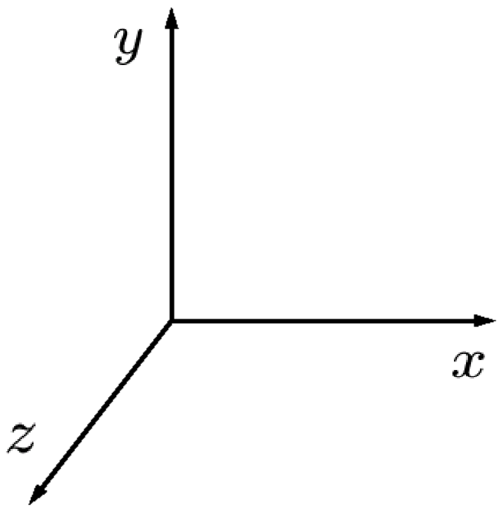






z-축을 중심으로 회전

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



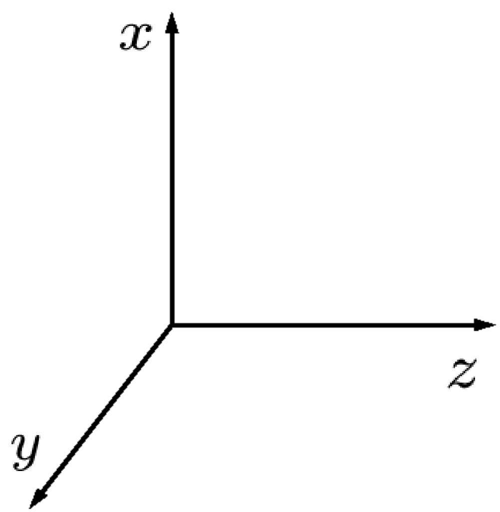
z-축을 중심으로 회전

$$\begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \\ z \\ 1 \end{bmatrix}$$

y-축을 중심으로 회전

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



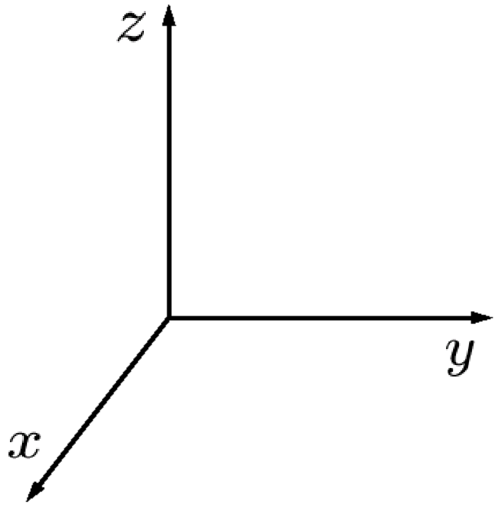
y-축을 중심으로 회전

$$\begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} x\cos\theta + z\sin\theta \\ y \\ -x\sin\theta + z\cos\theta \\ 1 \end{bmatrix}$$

x-축을 중심으로 회전

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



x-축을 중심으로 회전

$$\begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} x \\ y\cos\theta - z\sin\theta \\ y\sin\theta + z\cos\theta \\ 1 \end{bmatrix}$$

3차원 축소확대

$$\begin{bmatrix} \widehat{x} \\ \widehat{y} \\ \widehat{z} \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 \\ 0 & 0 & \gamma & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha x \\ \beta y \\ \gamma z \\ 1 \end{bmatrix}$$

3차원 평행이동

$$\begin{bmatrix} \widehat{x} \\ \widehat{y} \\ \widehat{z} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ z + t_z \\ 1 \end{bmatrix}$$

3차원 변환

$$\begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \\ 1 \end{bmatrix} = \begin{bmatrix} a_x & b_x & c_x & t_x \\ a_y & b_y & c_y & t_y \\ a_z & b_z & c_z & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} = \begin{bmatrix} a_x & b_x & c_x \\ a_y & b_y & c_y \\ a_z & b_z & c_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

3차원 변환

$$\begin{bmatrix} t_x \\ t_y \\ t_z \\ 1 \end{bmatrix} = \begin{bmatrix} a_x & b_x & c_x & t_x \\ a_y & b_y & c_y & t_y \\ a_z & b_z & c_z & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} a_x \\ a_y \\ a_z \\ 0 \end{bmatrix} = \begin{bmatrix} a_x & b_x & c_x & t_x \\ a_y & b_y & c_y & t_y \\ a_z & b_z & c_z & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

3차원 변환

$$\begin{bmatrix} b_x \\ b_y \\ b_z \\ 0 \end{bmatrix} = \begin{bmatrix} a_x & b_x & c_x & t_x \\ a_y & b_y & c_y & t_y \\ a_z & b_z & c_z & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

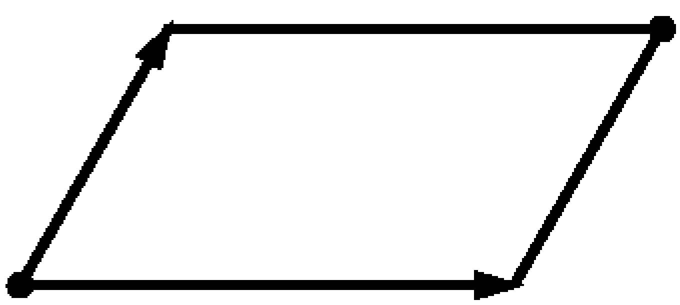
$$\begin{bmatrix} c_x \\ c_y \\ c_z \\ 0 \end{bmatrix} = \begin{bmatrix} a_x & b_x & c_x & t_x \\ a_y & b_y & c_y & t_y \\ a_z & b_z & c_z & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

행렬식

$$|a_{11}| = a_{11}$$

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

$$= a_{11}a_{22} - a_{21}a_{12}$$



행렬식

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

$$+ a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

행렬식

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{21} \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}$$

$$+ a_{31} \begin{vmatrix} a_{12} & a_{22} \\ a_{22} & a_{23} \end{vmatrix}$$

행렬식

$$\begin{vmatrix} 1 & 3 & 0 \\ 2 & 6 & 4 \\ -1 & 0 & 2 \end{vmatrix}$$

$$= 1\begin{vmatrix} 6 & 4 \\ 0 & 2 \end{vmatrix} - 3\begin{vmatrix} 2 & 4 \\ -1 & 2 \end{vmatrix} + 0\begin{vmatrix} 2 & 6 \\ -1 & 0 \end{vmatrix}$$

$$= 1(12 - 0) - 3(4 + 4)$$

$$= -12$$

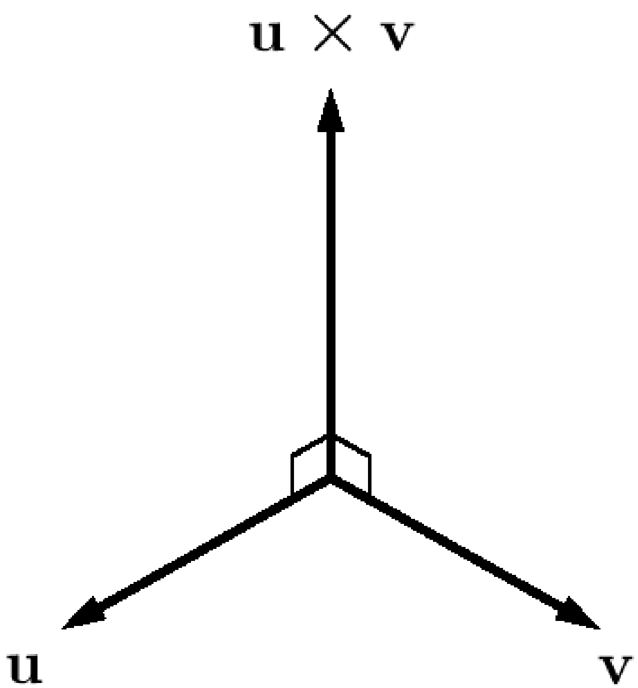
행렬식

$$\begin{vmatrix} -3 & 0 & 0 \\ 6 & 4 & 0 \\ -1 & 2 & 5 \end{vmatrix}$$

$$= -3 \begin{vmatrix} 4 & 0 \\ 2 & 5 \end{vmatrix}$$

$$= (-3) \cdot 4 \cdot 5 = -60$$

3차원 벡타의 외적



$$\mathbf{e}_1 = (1, 0, 0)$$
$$\mathbf{e}_2 = (0, 1, 0)$$
$$\mathbf{e}_3 = (0, 0, 1)$$

$$\begin{vmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3 \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

$$= \mathbf{e}_1 \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} - \mathbf{e}_2 \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix}$$

$$+\mathbf{e}_3 \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}$$

3차원 벡타의 외적

$$\begin{vmatrix} \mathrm{e}_1 & \mathrm{e}_2 & \mathrm{e}_3 \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

$$= \mathrm{e}_1 \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} - \mathrm{e}_2 \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} + \mathrm{e}_3 \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}$$

$$= (u_2v_3 - u_3v_2)\mathrm{e}_1 + (u_3v_1 - u_1v_3)\mathrm{e}_2$$

$$+(u_1v_2 - u_2v_1)\mathrm{e}_3$$

$$= (u_2v_3 - u_3v_2, u_3v_1 - u_1v_3, u_1v_2 - u_2v_1)$$

3차원 벡타의 외적

$$(1,1,0) \times (3,0,0)$$

$$= \begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & 1 & 0 \\ 3 & 0 & 0 \end{vmatrix}$$

$$= e_1 \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} - e_2 \begin{vmatrix} 1 & 0 \\ 3 & 0 \end{vmatrix} + e_3 \begin{vmatrix} 1 & 1 \\ 3 & 0 \end{vmatrix}$$

$$= (-3) \cdot e_3 = (0,0,-3)$$

벡타 외적의 기본성질

$$e_1 \times e_2 = e_3$$

$$e_2 \times e_3 = e_1$$

$$e_3 \times e_1 = e_2$$

$$e_2 \times e_1 = -e_3$$

$$e_3 \times e_2 = -e_1$$

$$e_1 \times e_3 = -e_2$$

벡타 외적의 기본성질

$$(k\mathbf{u}) \times \mathbf{v} = k(\mathbf{u} \times \mathbf{v})$$

$$\mathbf{u} \times (k\mathbf{v}) = k(\mathbf{u} \times \mathbf{v})$$

$$\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = \mathbf{u} \times \mathbf{v} + \mathbf{u} \times \mathbf{w}$$

$$(\mathbf{u} + \mathbf{v}) \times \mathbf{w} = \mathbf{u} \times \mathbf{w} + \mathbf{v} \times \mathbf{w}$$

$$\mathbf{u} \times \mathbf{v} = -\mathbf{v} \times \mathbf{u}$$

벡타 외적의 기본성질

$$\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) \neq (\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$$

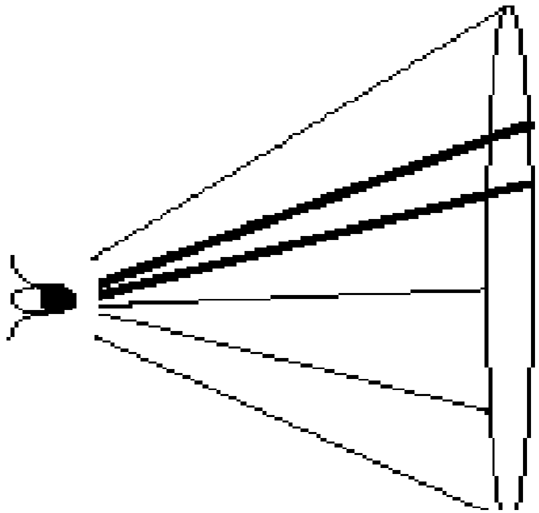
$$\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = \langle \mathbf{u}, \mathbf{w} \rangle \mathbf{v} - \langle \mathbf{u}, \mathbf{v} \rangle \mathbf{w}$$

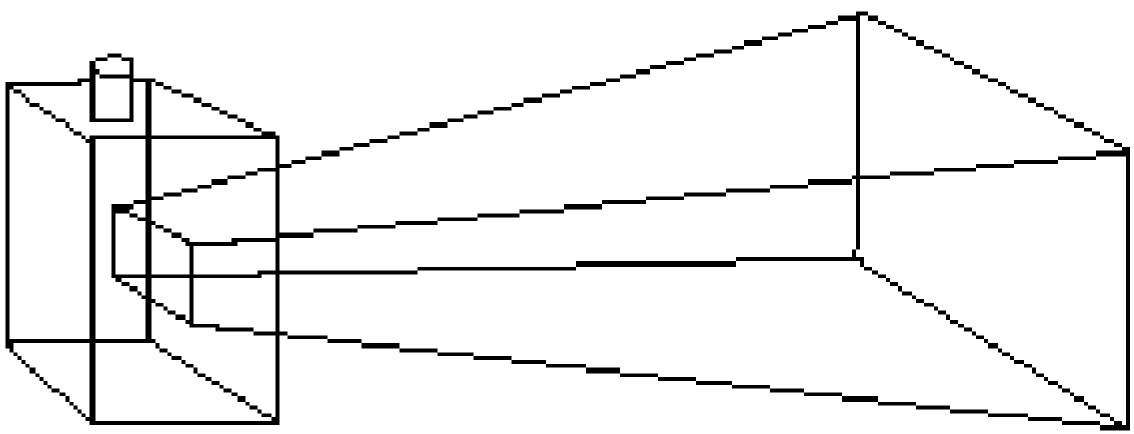
$$\langle \mathbf{u}, \mathbf{v} \times \mathbf{w} \rangle = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

3차원 Viewing 변환

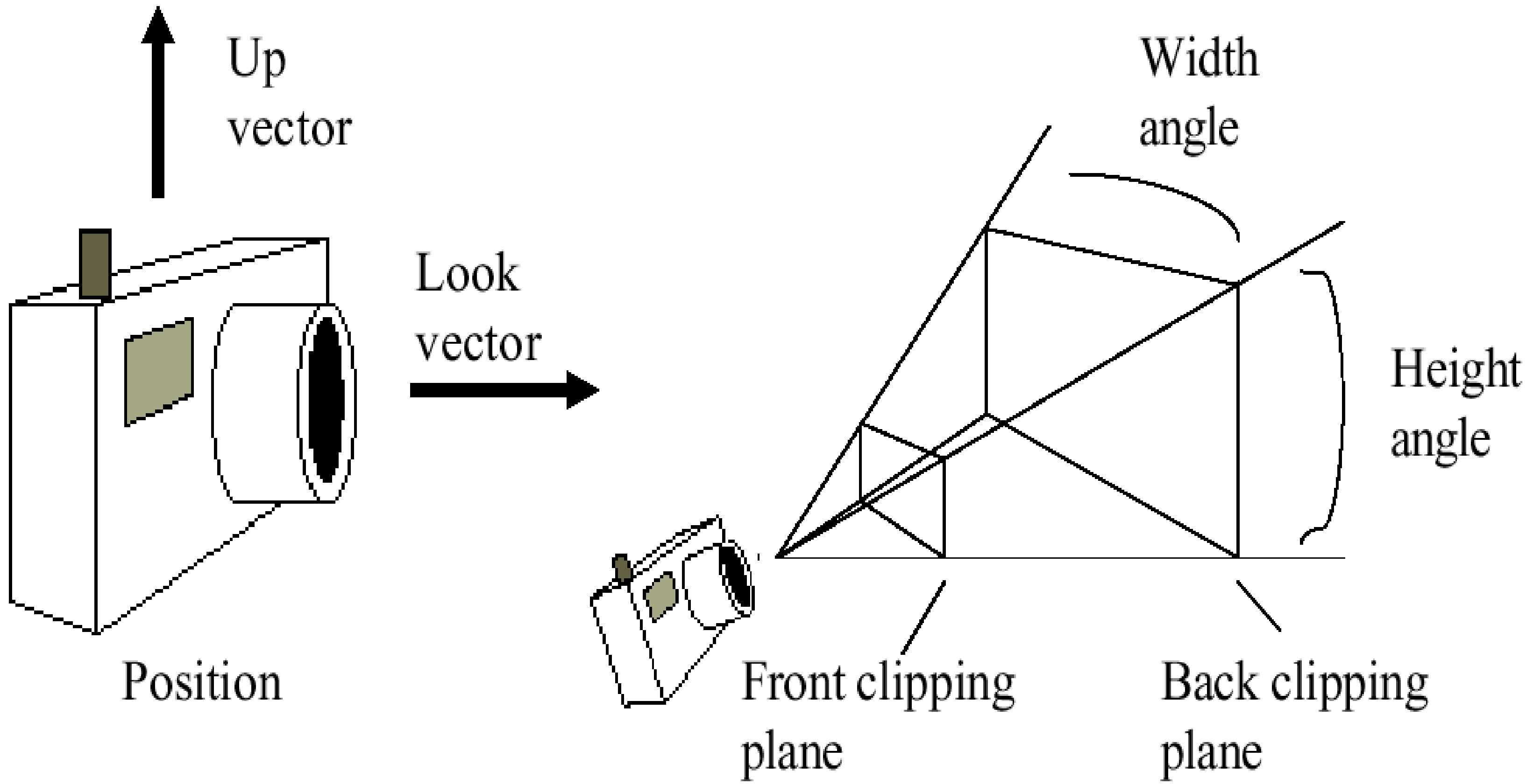
서울대학교 컴퓨터공학부
김명수
http://cse.snu.ac.kr/mskim
http://3map.snu.ac.kr

3차원 View Volume

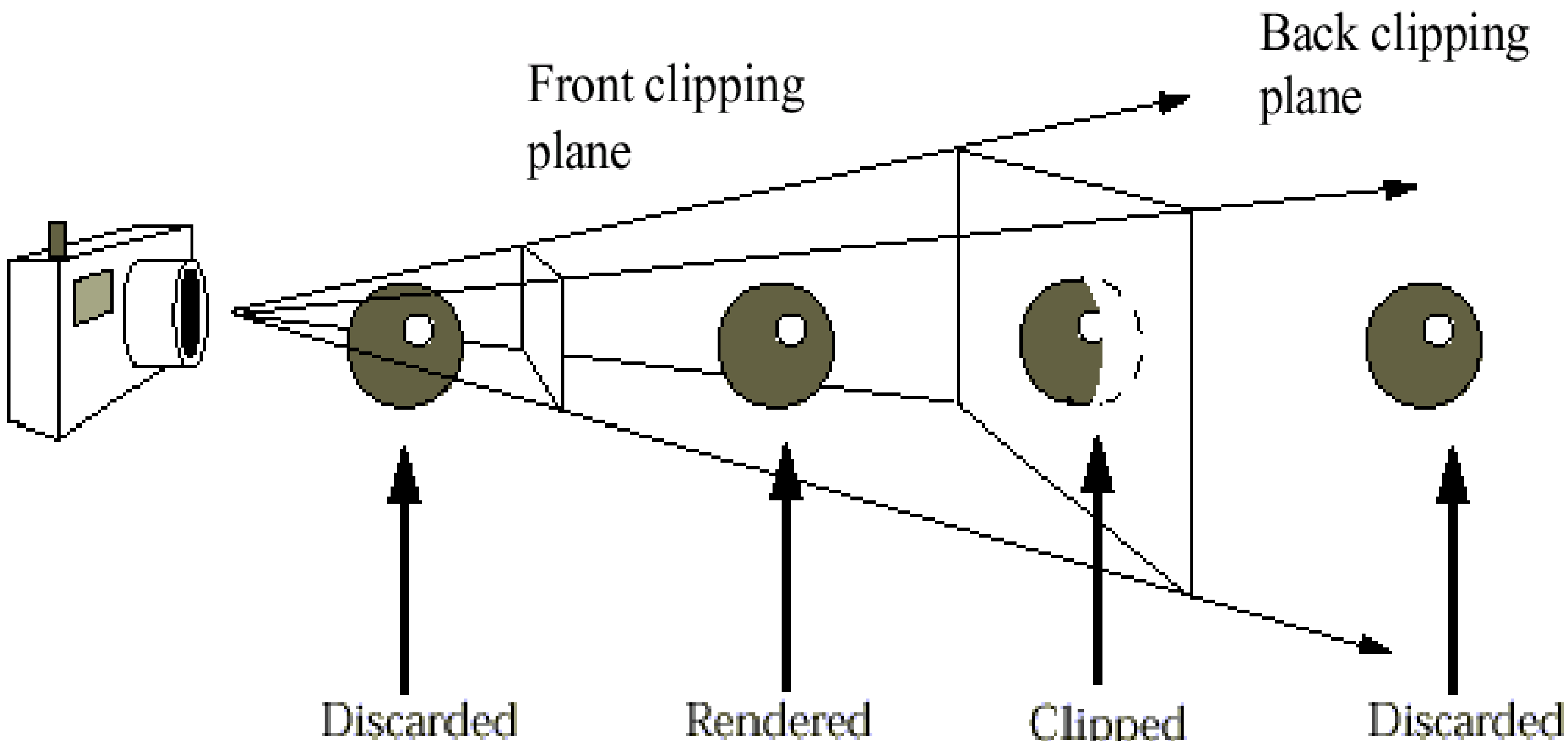




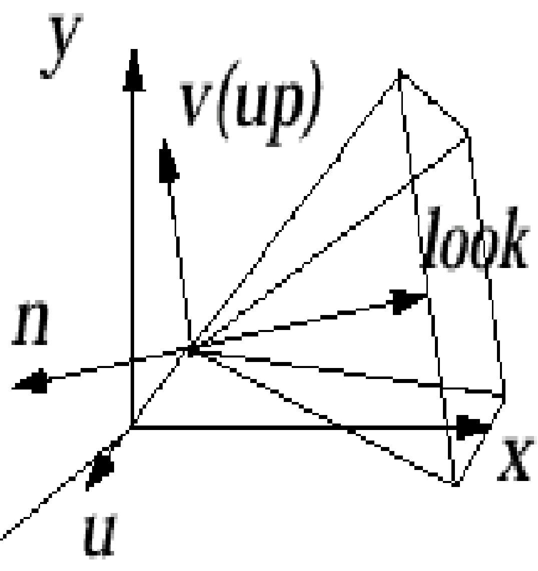
3차원 View Volume



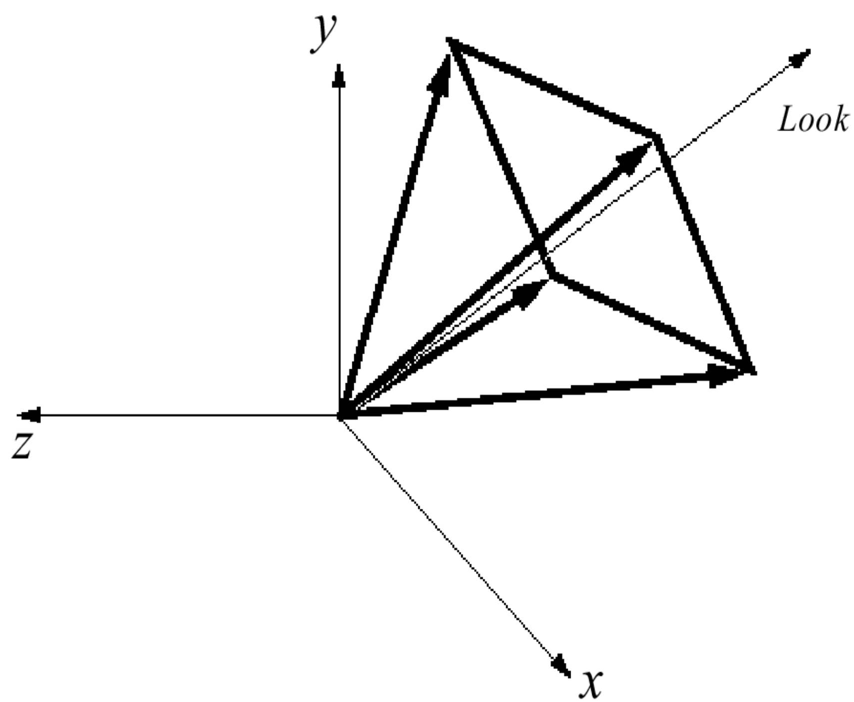
3차원 Clipping



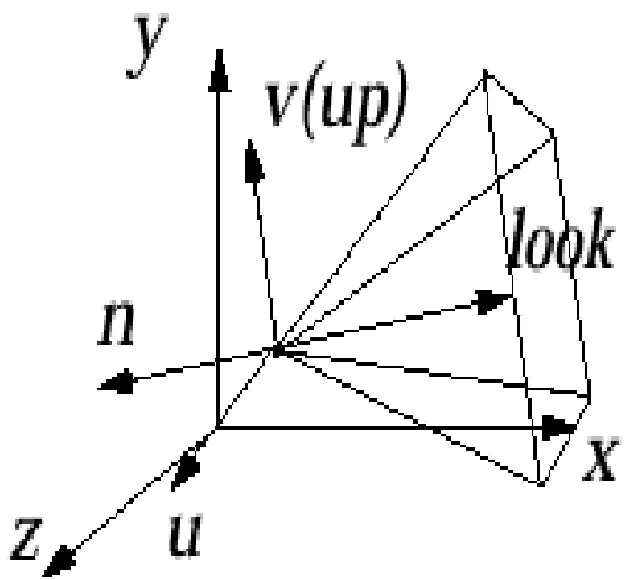
3차원 Viewing 변환



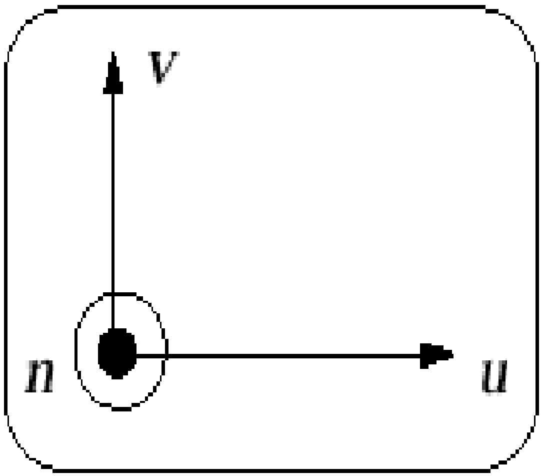
Arbitrary Perspective Frustum



3차원 Viewing 변환

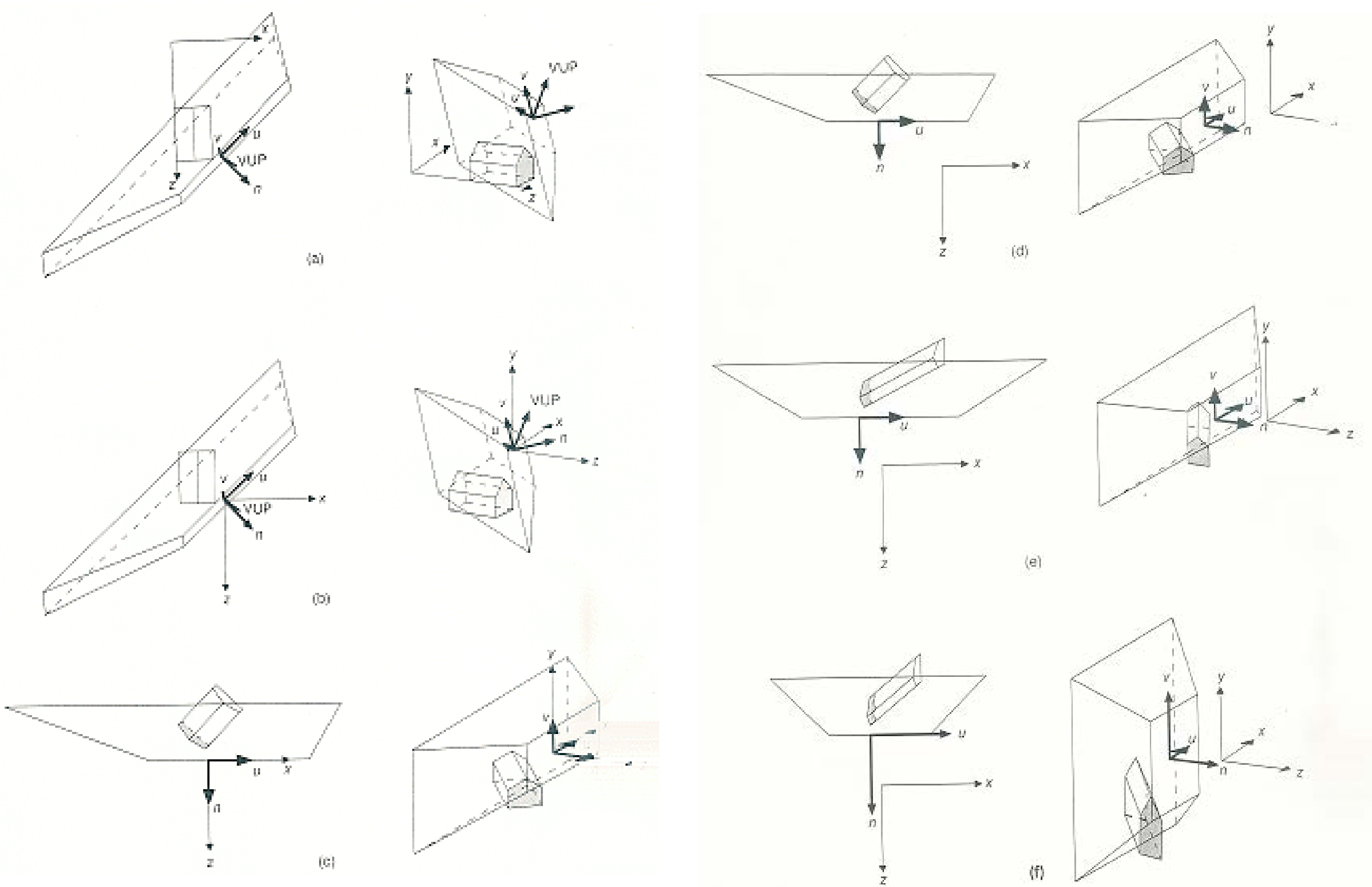


Arbitrary Perspective Frustum

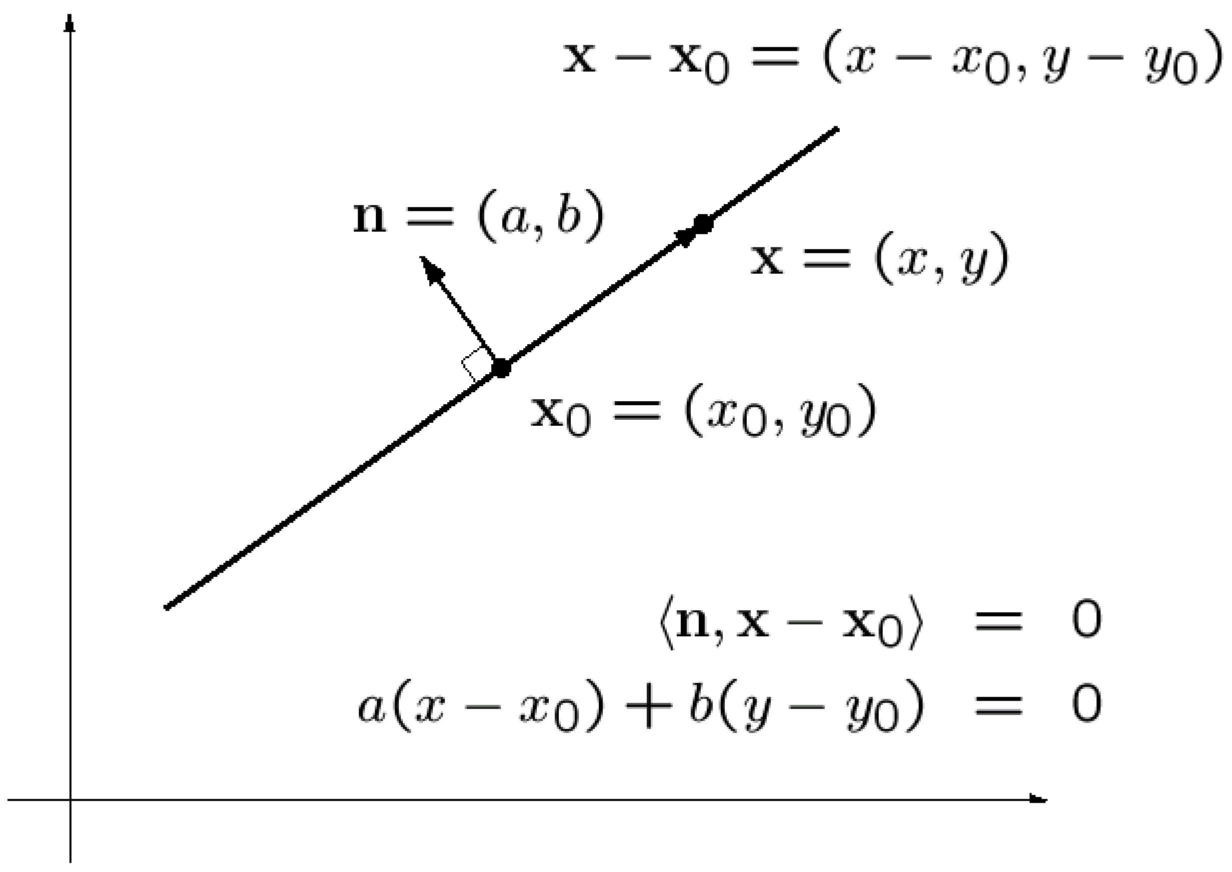


Camera Coordinate System
(with *n* coming out of the paper)

기존의 교과서에 나온 방법

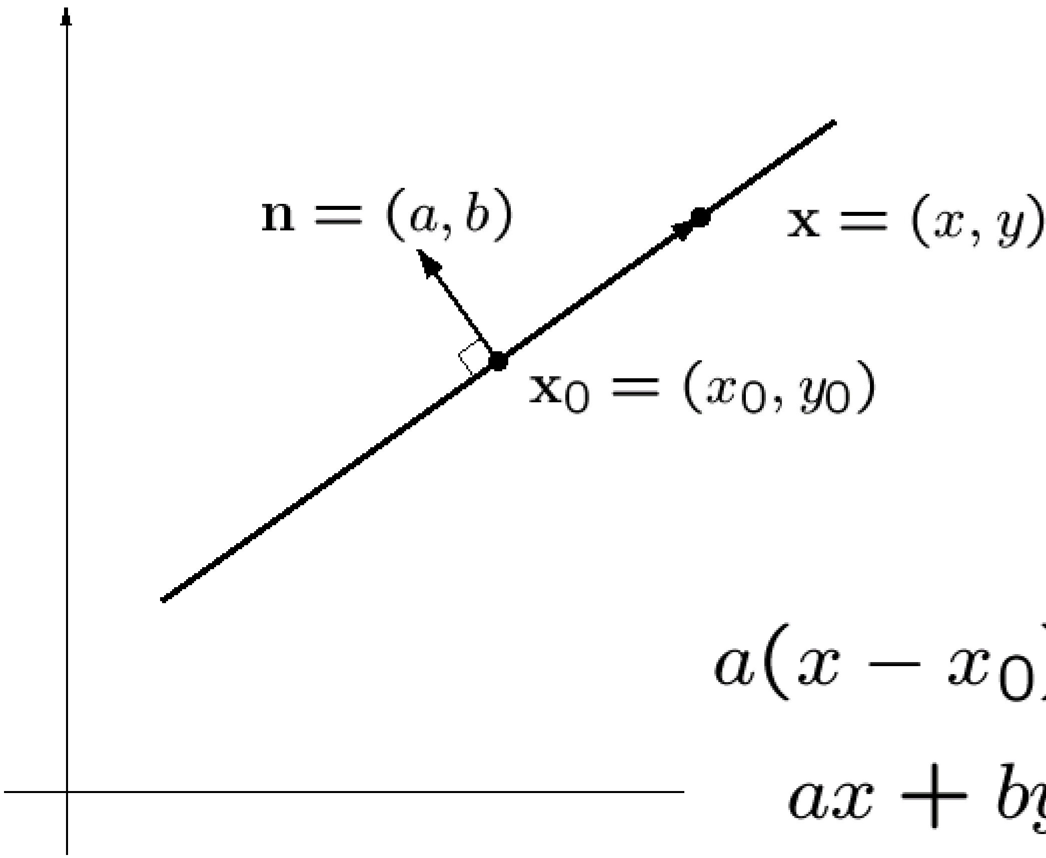


직선의 방정식



$\mathbf{x} - \mathbf{x}_0 = (x - x_0, y - y_0)$
$\mathbf{n} = (a, b)$
$\mathbf{x} = (x, y)$
$\mathbf{x}_0 = (x_0, y_0)$
$\langle \mathbf{n}, \mathbf{x} - \mathbf{x}_0 \rangle = 0$
$a(x - x_0) + b(y - y_0) = 0$

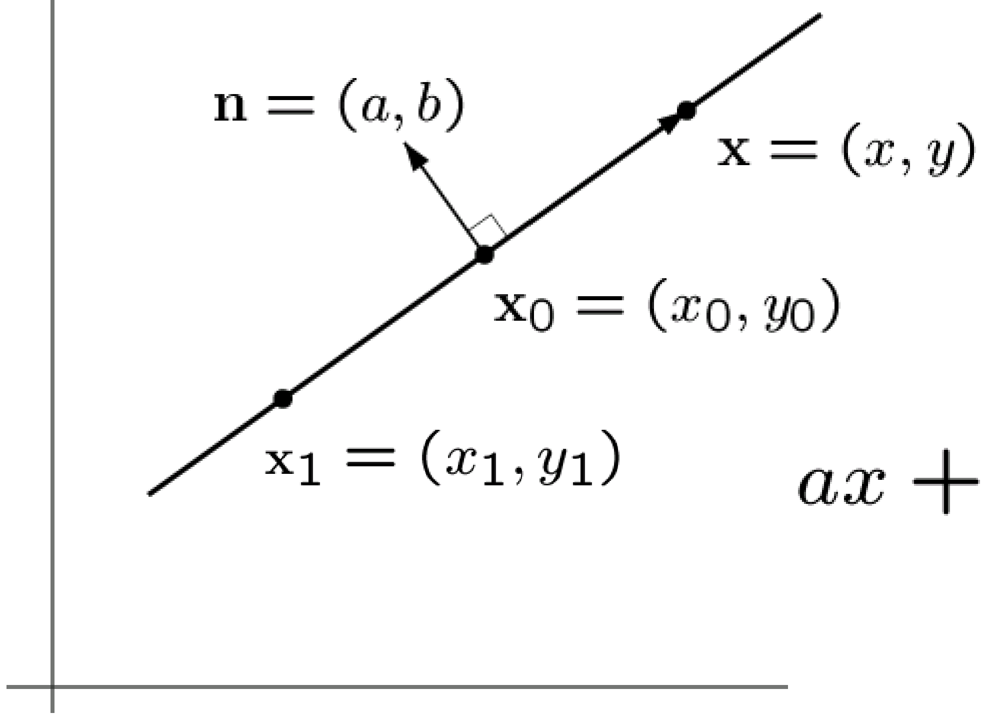
직선의 방정식



$$
\begin{aligned}
\langle \mathbf{n}, \mathbf{x} - \mathbf{x}_0 \rangle &= 0 \\
a(x - x_0) + b(y - y_0) &= 0 \\
ax + by - ax_0 - by_0 &= 0 \\
ax + by + c &= 0
\end{aligned}
$$

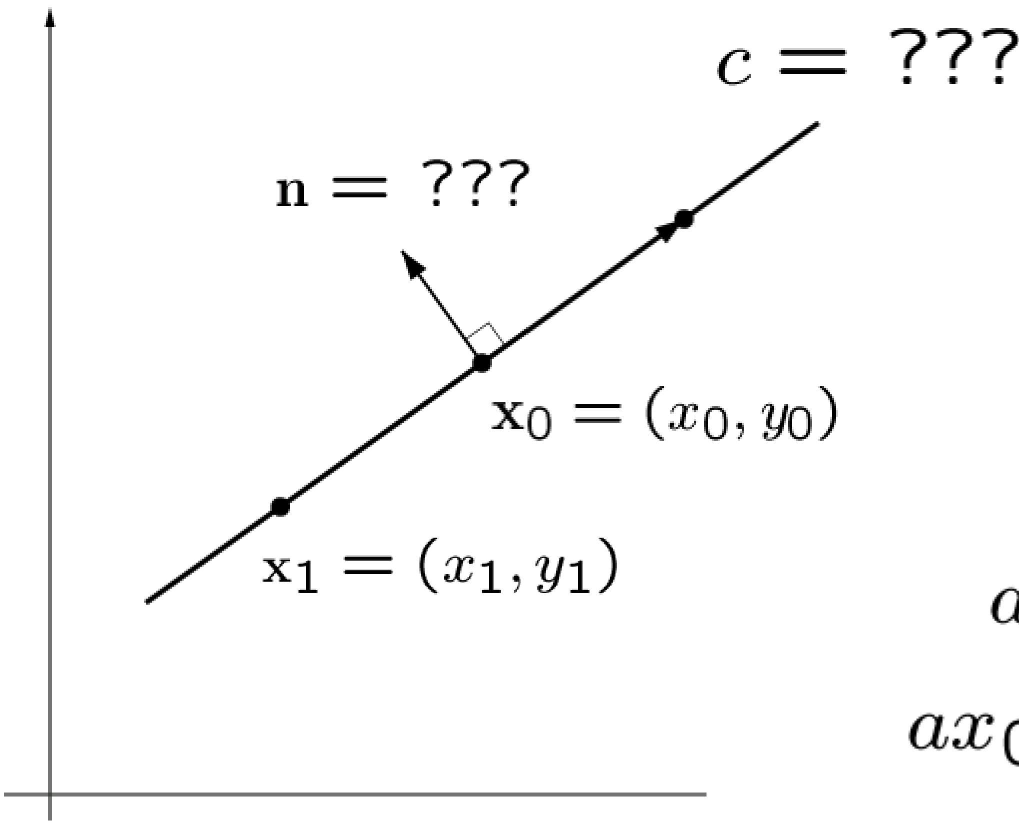
직선의 방정식

$$ax + by + c = 0$$



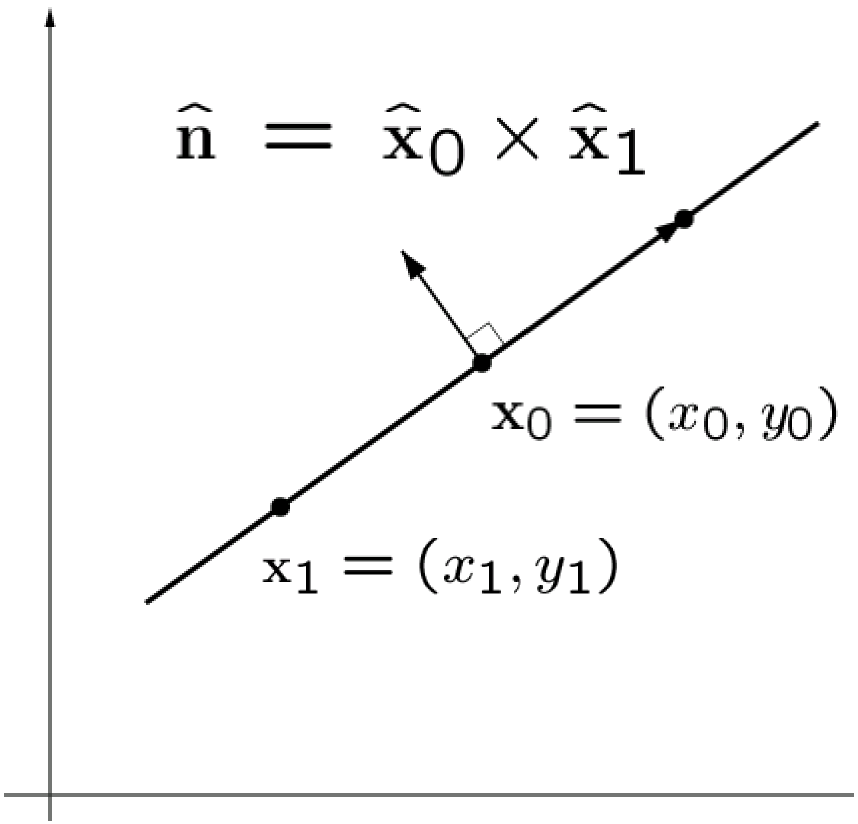
$$\begin{aligned} ax + by - ax_0 - by_0 &= 0 \\ ax + by + c &= 0 \\ ax_0 + by_0 + c \cdot 1 &= 0 \\ ax_1 + by_1 + c \cdot 1 &= 0 \end{aligned}$$

두점으로 부터 직선구하기



$$ax + by + c \cdot 1 = 0$$
$$ax_0 + by_0 + c \cdot 1 = 0$$
$$ax_1 + by_1 + c \cdot 1 = 0$$

두점으로 부터 직선구하기

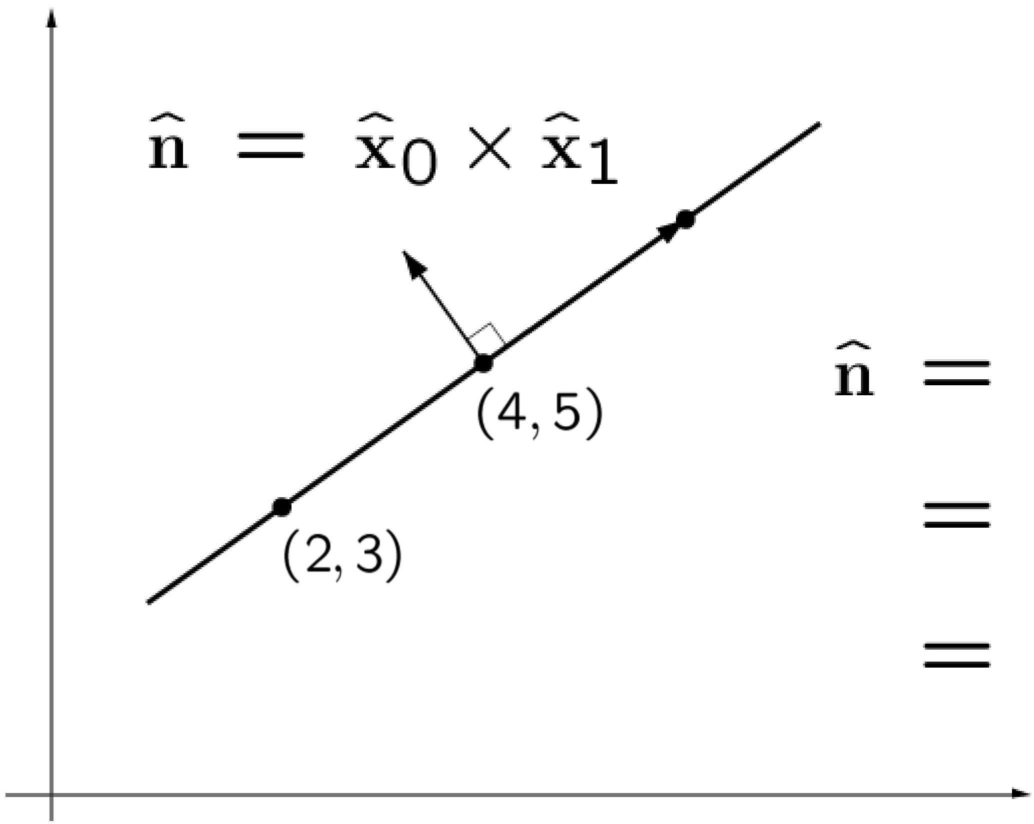


$$
\begin{aligned}
\hat{\mathbf{n}} &= (a, b, c) \\
\hat{\mathbf{x}}_0 &= (x_0, y_0, 1) \\
\hat{\mathbf{x}}_1 &= (x_1, y_1, 1)
\end{aligned}
$$

$$
\begin{aligned}
ax_0 + by_0 + c \cdot 1 &= 0 \\
ax_1 + by_1 + c \cdot 1 &= 0 \\
\langle \hat{\mathbf{n}}, \hat{\mathbf{x}}_0 \rangle &= 0 \\
\langle \hat{\mathbf{n}}, \hat{\mathbf{x}}_1 \rangle &= 0
\end{aligned}
$$

두점으로 부터 직선구하기

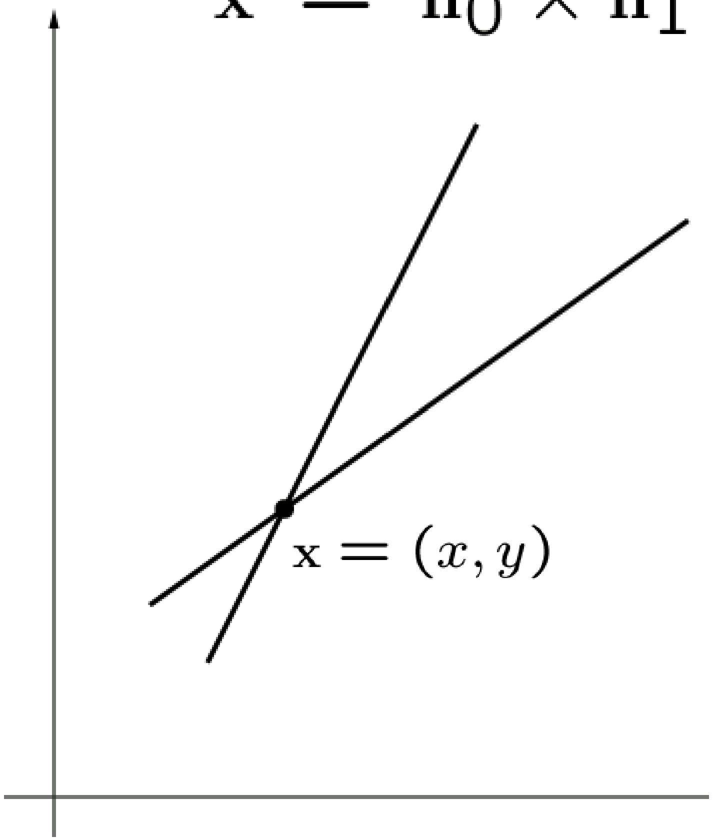
$$\hat{\mathbf{x}}_0 = (2,3,1)$$
$$\hat{\mathbf{x}}_1 = (4,5,1)$$



$$\begin{aligned}\hat{\mathbf{n}} &= \hat{\mathbf{x}}_0 \times \hat{\mathbf{x}}_1 \\ &= (2,3,1) \times (4,5,1) \\ &= (-2,2,-2)\end{aligned}$$

두직선으로 부터 교점구하기

$$\hat{\mathbf{x}} = \hat{\mathbf{n}}_0 \times \hat{\mathbf{n}}_1$$



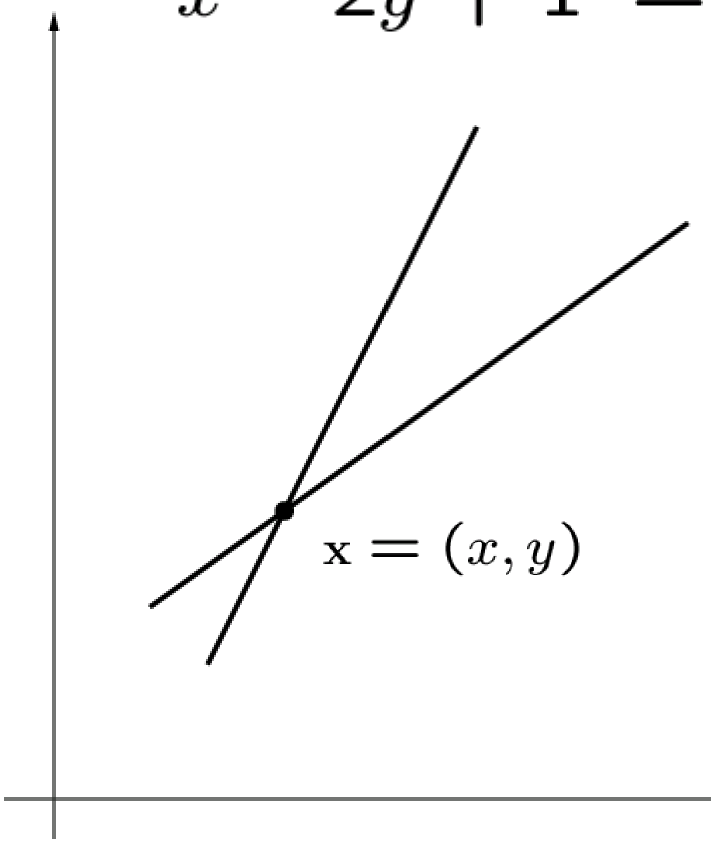
$$\hat{\mathbf{x}} = (x, y, 1)$$
$$\hat{\mathbf{n}}_0 = (a_0, b_0, c_0)$$
$$\hat{\mathbf{n}}_1 = (a_1, b_1, c_1)$$

$$a_0 \cdot x + b_0 \cdot y + c_0 \cdot 1 = 0$$
$$a_1 \cdot x + b_1 \cdot y + c_1 \cdot 1 = 0$$
$$\langle \hat{\mathbf{n}}_0, \hat{\mathbf{x}} \rangle = 0$$
$$\langle \hat{\mathbf{n}}_1, \hat{\mathbf{x}} \rangle = 0$$

두직선으로 부터 교점구하기

$$x + y - 1 = 0$$
$$x - 2y + 1 = 0$$

$$\hat{\mathbf{x}} = (x, y, 1)$$
$$\hat{\mathbf{n}}_0 = (1, 1, -1)$$
$$\hat{\mathbf{n}}_1 = (1, -2, 1)$$



$$\begin{aligned}\hat{\mathbf{x}} &= \hat{\mathbf{n}}_0 \times \hat{\mathbf{n}}_1 \\ &= (1, 1, -1) \times (1, -2, 1) \\ &= (-1, -2, -3) \\ &= \left(\frac{1}{3}, \frac{2}{3}, 1\right)\end{aligned}$$

평면의 방정식

$$\begin{aligned}\mathbf{n} &= (a, b, c) \\ \mathbf{x} - \mathbf{x}_0 &= (x - x_0, y - y_0, z - z_0)\end{aligned}$$

$$\begin{aligned}0 &= \langle \mathbf{n}, \mathbf{x} - \mathbf{x}_0 \rangle \\ 0 &= a(x - x_0) + b(y - y_0) + c(z - z_0) \\ 0 &= ax + by + cz - ax_0 - by_0 - cz_0 \\ 0 &= ax + by + cz + d\end{aligned}$$

세점으로 부터 평면구하기

$$
\begin{aligned}
\hat{\mathbf{n}} &= (a, b, c, d) \\
\hat{\mathbf{x}}_0 &= (x_0, y_0, z_0, 1) \\
\hat{\mathbf{x}}_1 &= (x_1, y_1, z_1, 1) \\
\hat{\mathbf{x}}_2 &= (x_2, y_2, z_2, 1)
\end{aligned}
$$

$$
\begin{aligned}
ax_0 + by_0 + cz_0 + d \cdot 1 &= 0 \\
ax_1 + by_1 + cz_1 + d \cdot 1 &= 0 \\
ax_2 + by_2 + cz_2 + d \cdot 1 &= 0
\end{aligned}
$$

세점으로 부터 평면구하기

$$
\begin{aligned}
ax_0 + by_0 + cz_0 + d \cdot 1 &= 0 \\
ax_1 + by_1 + cz_1 + d \cdot 1 &= 0 \\
ax_2 + by_2 + cz_2 + d \cdot 1 &= 0
\end{aligned}
$$

$$
\begin{aligned}
\langle \widehat{\mathbf{n}}, \widehat{\mathbf{x}}_0 \rangle &= 0 \\
\langle \widehat{\mathbf{n}}, \widehat{\mathbf{x}}_1 \rangle &= 0 \\
\langle \widehat{\mathbf{n}}, \widehat{\mathbf{x}}_2 \rangle &= 0 \\
\widehat{\mathbf{n}} &= \widehat{\mathbf{x}}_0 \wedge \widehat{\mathbf{x}}_1 \wedge \widehat{\mathbf{x}}_2
\end{aligned}
$$

Wedge Product

$$
\begin{aligned}
\hat{\mathbf{n}} &= \hat{\mathbf{x}}_0 \wedge \hat{\mathbf{x}}_1 \wedge \hat{\mathbf{x}}_2 \\
&= \begin{vmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3 & \mathbf{e}_4 \\ x_0 & y_0 & z_0 & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \end{vmatrix}
\end{aligned}
$$

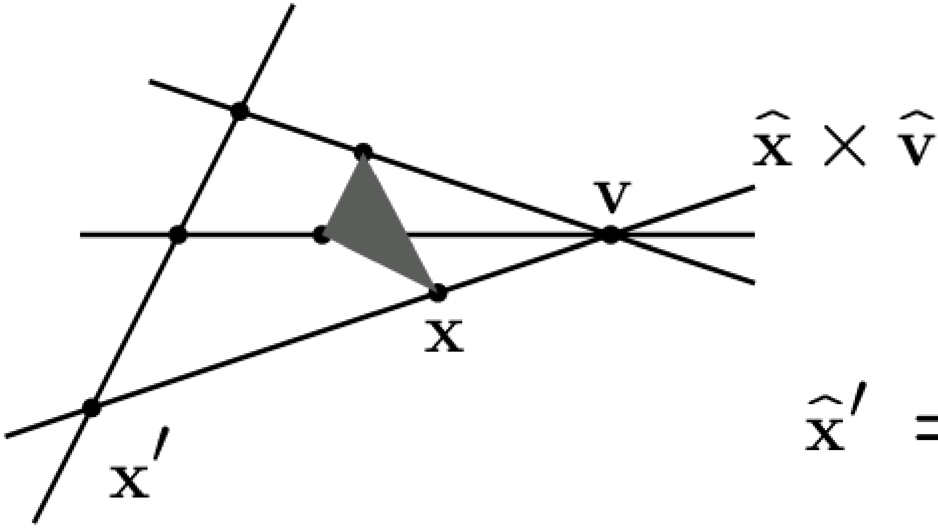
세평면으로 부터 교점구하기

$$
\begin{aligned}
\hat{\mathbf{x}} &= (x, y, z, w) \\
\hat{\mathbf{n}}_0 &= (a_0, b_0, c_0, d_0) \\
\hat{\mathbf{n}}_1 &= (a_1, b_1, c_1, d_1) \\
\hat{\mathbf{n}}_2 &= (a_2, b_2, c_2, d_2)
\end{aligned}
$$

$$\hat{\mathbf{x}} = \hat{\mathbf{n}}_0 \wedge \hat{\mathbf{n}}_1 \wedge \hat{\mathbf{n}}_2$$

2차원에서의 투영변환

$$\hat{\mathbf{n}} = (a, b, c)$$



$$\hat{\mathbf{x}}' = (x', y', 1)$$
$$= \hat{\mathbf{n}} \times (\hat{\mathbf{x}} \times \hat{\mathbf{v}})$$
$$= \langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle \hat{\mathbf{x}} - \langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle \hat{\mathbf{v}}$$
$$= \langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle \hat{\mathbf{x}} - \hat{\mathbf{v}} \langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle$$

$$\hat{\mathbf{x}} = (x, y, 1)$$
$$\hat{\mathbf{v}} = (v_x, v_y, 1)$$

2차원에서의 투영변환

$$\widehat{\mathbf{x}}' = \langle \widehat{\mathbf{n}}, \widehat{\mathbf{v}} \rangle \widehat{\mathbf{x}} - \widehat{\mathbf{v}} \langle \widehat{\mathbf{n}}, \widehat{\mathbf{x}} \rangle$$

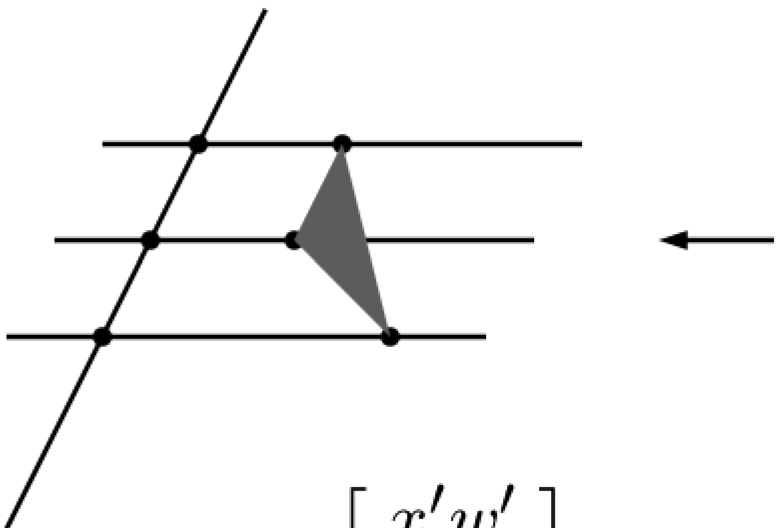
$$\begin{bmatrix} x'w' \\ y'w' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$- \begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} bv_y + c & -bv_x & -cv_x \\ -av_y & av_x + c & -cv_y \\ -a & -b & av_x + bv_y \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2차원에서의 평행투영변환

$$\widehat{\mathbf{x}}' = \langle \widehat{\mathbf{n}}, \widehat{\mathbf{v}} \rangle \widehat{\mathbf{x}} - \widehat{\mathbf{v}} \langle \widehat{\mathbf{n}}, \widehat{\mathbf{x}} \rangle$$



$$\hat{\mathbf{v}} = (v_x, v_y, 0)$$

$$\begin{bmatrix} x'w' \\ y'w' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$- \begin{bmatrix} v_x \\ v_y \\ 0 \end{bmatrix} \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2차원에서의 평행투영변환

$$\hat{\mathbf{x}}' = \langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle \hat{\mathbf{x}} - \hat{\mathbf{v}} \langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle$$

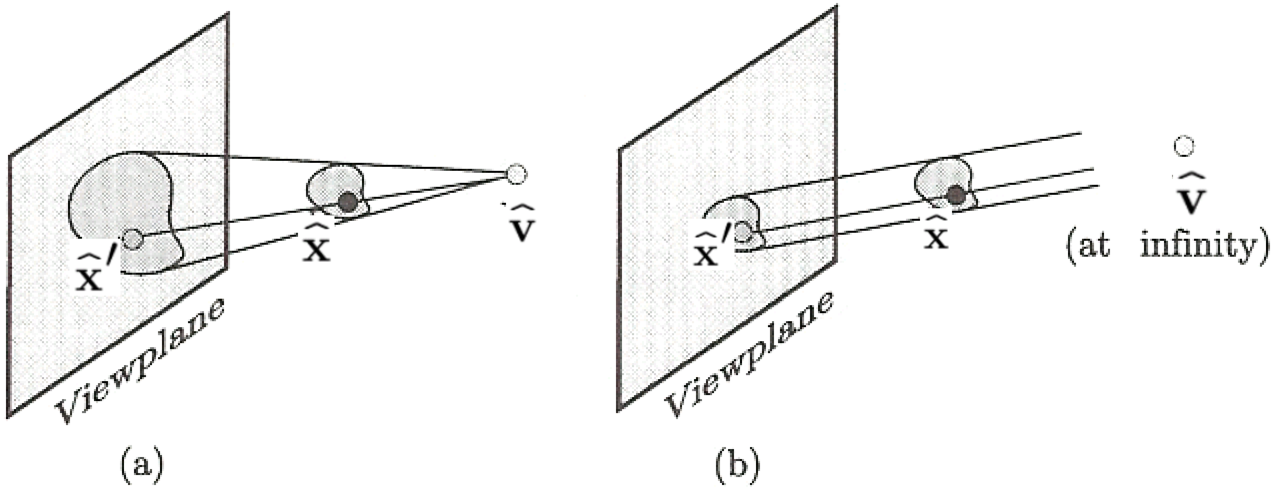
$$\begin{bmatrix} x'w' \\ y'w' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$- \begin{bmatrix} v_x \\ v_y \\ 0 \end{bmatrix} \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} bv_y & -bv_x & -cv_x \\ -av_y & av_x & -cv_y \\ 0 & 0 & av_x + bv_y \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

3차원에서의 투영변환

$$\widehat{\mathbf{x}}' = \langle \widehat{\mathbf{n}}, \widehat{\mathbf{v}} \rangle \widehat{\mathbf{x}} - \widehat{\mathbf{v}} \langle \widehat{\mathbf{n}}, \widehat{\mathbf{x}} \rangle$$



Perspective and parallel three-dimensional projections

3차원에서의 투영변환

$$\hat{\mathbf{x}}' = \langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle \hat{\mathbf{x}} - \hat{\mathbf{v}} \langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle$$

(Case I): $\langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle = 0$,

$$\hat{\mathbf{x}}' = \langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle \hat{\mathbf{x}} = \hat{\mathbf{x}}$$

(Case II): $\langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle \neq 0$,

$$\begin{aligned} \hat{\mathbf{x}}' &= \alpha \hat{\mathbf{x}} + \beta \hat{\mathbf{v}} \\ 0 &= \langle \hat{\mathbf{n}}, \hat{\mathbf{x}}' \rangle \\ 0 &= \langle \hat{\mathbf{n}}, \alpha \hat{\mathbf{x}} + \beta \hat{\mathbf{v}} \rangle \end{aligned}$$

3차원에서의 투영변환

$$\hat{\mathbf{x}}' = \langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle \hat{\mathbf{x}} - \hat{\mathbf{v}} \langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle$$

(Case II): $\langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle \neq 0$,

$$\begin{aligned}
\hat{\mathbf{x}}' &= \alpha \hat{\mathbf{x}} + \beta \hat{\mathbf{v}} \\
0 &= \langle \hat{\mathbf{n}}, \hat{\mathbf{x}}' \rangle \\
0 &= \langle \hat{\mathbf{n}}, \alpha \hat{\mathbf{x}} + \beta \hat{\mathbf{v}} \rangle \\
0 &= \alpha \langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle + \beta \langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle \\
\alpha &= -\beta \frac{\langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle}{\langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle}
\end{aligned}$$

3차원에서의 투영변환

$$\hat{\mathbf{x}}' = \langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle \hat{\mathbf{x}} - \hat{\mathbf{v}} \langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle$$

(Case II): $\langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle \neq 0$,

$$\alpha = -\beta \frac{\langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle}{\langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle}$$

$$\begin{aligned} \hat{\mathbf{x}}' &= -\beta \frac{\langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle}{\langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle} \hat{\mathbf{x}} + \beta \hat{\mathbf{v}} \\ &= \frac{\langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle}{\langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle} \hat{\mathbf{x}} - \hat{\mathbf{v}} \\ &= \langle \hat{\mathbf{n}}, \hat{\mathbf{v}} \rangle \hat{\mathbf{x}} - \langle \hat{\mathbf{n}}, \hat{\mathbf{x}} \rangle \hat{\mathbf{v}} \end{aligned}$$

3차원에서의 투영변환

$$\widehat{\mathbf{x}}' = \langle \widehat{\mathbf{n}}, \widehat{\mathbf{v}} \rangle \widehat{\mathbf{x}} - \widehat{\mathbf{v}} \langle \widehat{\mathbf{n}}, \widehat{\mathbf{x}} \rangle$$

$$\begin{bmatrix} x'w' \\ y'w' \\ z'w' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$- \begin{bmatrix} v_x \\ v_y \\ v_z \\ 1 \end{bmatrix} \begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3차원에서의 평행투영변환

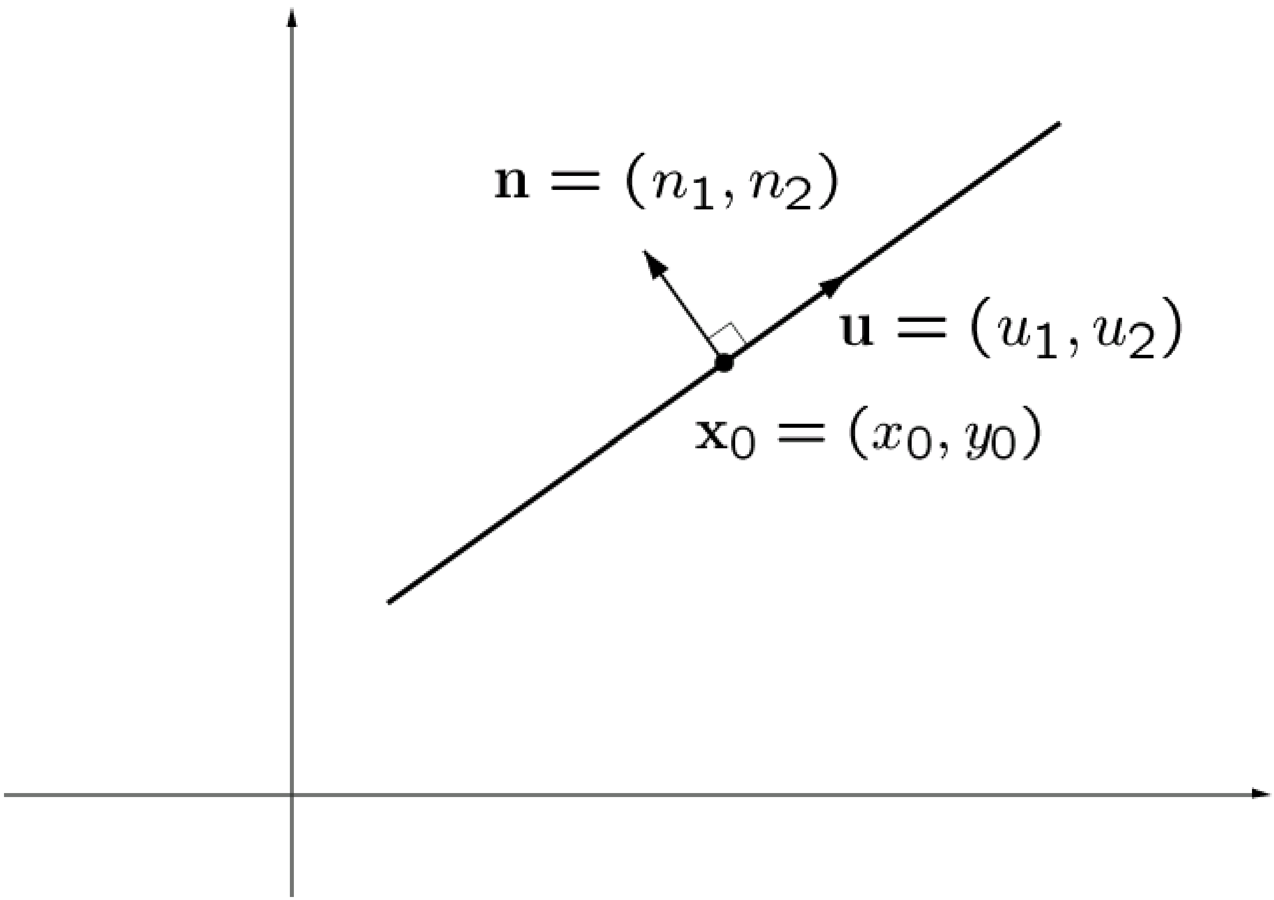
$$\widehat{\mathbf{x}}' = \langle \widehat{\mathbf{n}}, \widehat{\mathbf{v}} \rangle \widehat{\mathbf{x}} - \widehat{\mathbf{v}} \langle \widehat{\mathbf{n}}, \widehat{\mathbf{x}} \rangle$$

$$\begin{bmatrix} x'w' \\ y'w' \\ z'w' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$- \begin{bmatrix} v_x \\ v_y \\ v_z \\ 0 \end{bmatrix} \begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

2차원에서의 Viewing 변환

$$\|\mathbf{u}\| = \|\mathbf{n}\| = 1$$



$$\begin{bmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} u_1 & u_2 & 0 \\ n_1 & n_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2차원에서의 Viewing 변환

$$\begin{bmatrix} u_1 & u_2 & 0 \\ n_1 & n_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} u_1 & u_2 & 0 \\ n_1 & n_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} u_1 & u_2 & 0 \\ n_1 & n_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

3차원에서의 Viewing 변환

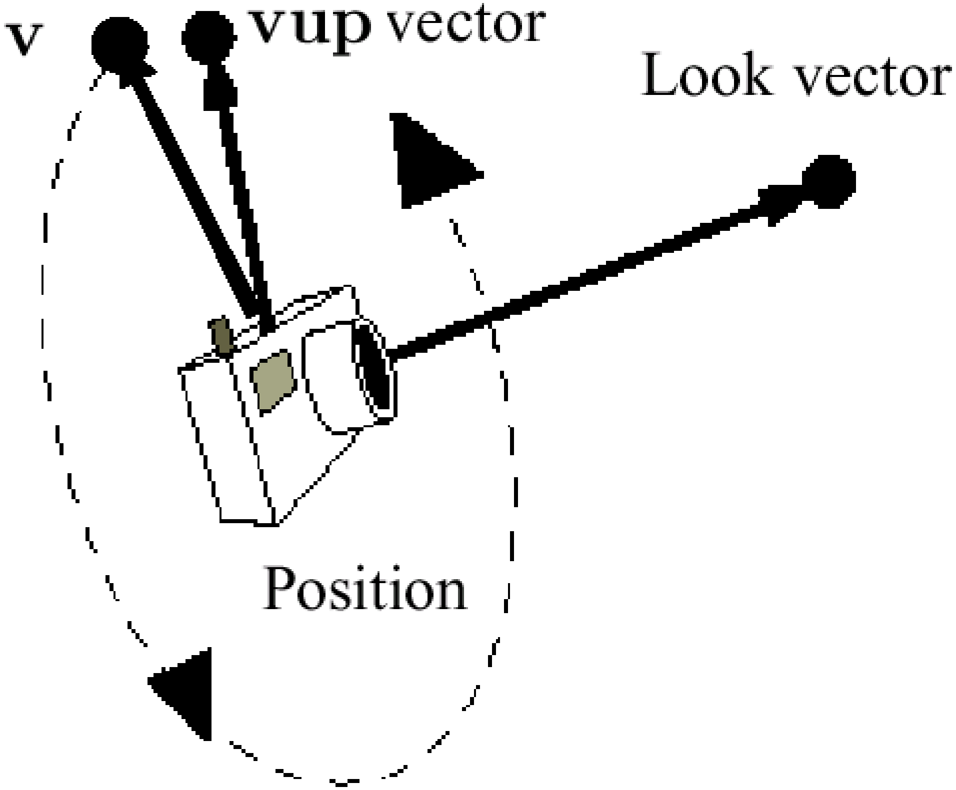
$$\begin{aligned}\mathbf{x} &= (x_0, y_0, z_0)\\ \mathbf{u} &= (u_1, u_2, u_3)\\ \mathbf{v} &= (v_1, v_2, v_3)\\ \mathbf{n} &= (n_1, n_2, n_3)\end{aligned}$$

$$\langle \mathbf{u}, \mathbf{v} \rangle = \langle \mathbf{u}, \mathbf{n} \rangle = \langle \mathbf{v}, \mathbf{n} \rangle = 0$$

$$\|\mathbf{u}\| = \|\mathbf{v}\| = \|\mathbf{n}\| = 1$$

$$\begin{bmatrix} u_1 & u_2 & u_3 & 0 \\ v_1 & v_2 & v_3 & 0 \\ n_1 & n_2 & n_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -x_0 \\ 0 & 1 & 0 & -y_0 \\ 0 & 0 & 1 & -z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3차원에서의 Viewing 변환



$$\mathbf{u} = \frac{\mathbf{vup} \times \mathbf{n}}{\|\mathbf{vup} \times \mathbf{n}\|}$$

$$\mathbf{v} = \mathbf{n} \times \mathbf{u}$$

3차원에서의 Viewing 변환

- 3차원 상의 점들을 2차원 viewing 평면으로 투영
- Viewing 평면상의 기준점 (x0,y0,z0)을 원점으로 이동
- U, V, N 방향을 X, Y, Z의 좌표계의 방향으로 회전
- 결과적으로 XY 평면상에 투영된 그림이 나타난다.