Quiz #2 (CSE 4190.313)

Wednesday, April 2, 2014

Name: _____   **ID No:** _____

1. (10 points) Suppose the matrices in $PA = LU$ are

$$
\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}
\begin{bmatrix} 0 & 0 & 1 & -3 & 2 \\ 2 & -1 & 4 & 2 & 1 \\ 4 & -2 & 9 & 1 & 4 \\ 2 & -1 & 5 & -1 & 5 \end{bmatrix}
=
\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix}
\begin{bmatrix} 2 & -1 & 4 & 2 & 1 \\ 0 & 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
$$

(a) (5 point) What is a basis for the nullspace of A?

(b) (5 point) What is a basis for the left nullspace of A?

2. (4 points)

    (a) (1 point) Construct a matrix whose nullspace contains the vector $\mathbf{x} = (1, 1, 2)$.

    (b) (1 point) Construct a matrix whose left nullspace contains $\mathbf{y} = (1, 5)$.

    (c) (2 points) Construct a matrix whose column space is spanned by $(1, 1, 2)$ and whose row space is spanned by $(1, 5)$.

3. (6 points)

    (a) (3 point) Show that the product ST of two reflections is a rotation:

$$ST = \begin{bmatrix} \cos 2\beta & \sin 2\beta \\ \sin 2\beta & -\cos 2\beta \end{bmatrix} \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{bmatrix}$$

    (b) (3 points) Suppose T is reflection across the $45°$ line, and S is reflection across the y-axis. Find the matrix represenations for ST and TS and interpret them as two different rotations.