Quiz #4 (CSE 4190.313)

Wednesday 28, 2014

Name: ______________________ **ID No:** ______________________

1. (5 points) Test to see if $A^T A$ is positive definite in each case:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 2 \\ 3 & 3 \\ 2 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}$$

2. (10 points) If $A = Q\Lambda Q^T$ is symmetric positive definite, then $R = Q\sqrt{\Lambda}Q^T$ is its symmetric positive definite square root. Compute R and verify $R^2 = A$ for

$$A = \begin{bmatrix} 10 & 6 \\ 6 & 10 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 10 & -6 \\ -6 & 10 \end{bmatrix}.$$

3. (15 points) For the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 2 & 1 \end{bmatrix}$,

(a) (10 points) Compute $A^T A$ and AA^T, and their eigenvalues and unit eigenvectors.

(b) (5 points) Multiply the three matrices $U\Sigma V^T$ to recover A.

4. (5 points) For any symmetric matrix A, compute the ratio $R(\mathbf{x})$ for the special choice $\mathbf{x} = (1, \cdots, 1)$. How is the sum $\Sigma_{i=1}^n \Sigma_{j=1}^n a_{ij}$ related to λ_1 and λ_n?

5. (5 points) If B is positive definite, show that the smallest eigenvalue of $A + B$ is larger than the smallest eigenvalue of A.