Linear and Nonlinear Computation Models
(CSE 4190.313)

Midterm Exam: April 20, 2015

Problem	Score
1	
2	
3	
4	
5	
6	
7	
Total	

Name: _____

ID No: _____

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1. (10 points) The matrix P that multiplies $(x, y, z)^T$ to give $(z, x, y)^T$ is a rotation matrix.

   (a) (4 points) Find P and P^3.

   (b) (6 points) The rotation axis $\mathbf{a} = (1, 1, 1)^T$ doesn't move: $P\mathbf{a} = \mathbf{a}$. What is the angle of rotation from $\mathbf{v} = (2, 3, -5)^T$ to $P\mathbf{v} = (-5, 2, 3)^T$?

2. (15 points) If A and B are square matrices, show that $I - BA$ is invertible if $I - AB$ is invertible.

3. (15 points)

(a) (4 points) If $A\mathbf{x} = \mathbf{b}$ always has at least one solution, show that the only solution to $A^T\mathbf{y} = \mathbf{0}$ is $\mathbf{y} = \mathbf{0}$.

(b) (4 points) For an $m \times n$ matrix A of rank r, suppose $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions for every $\mathbf{b}$. What are the conditions on the numbers m, n, and r?

(c) (7 points) Given an invertible 3×3 matrix A, write bases for the four fundamental subspaces for the 3×6 matrix $B = [A\ A]$.

4. (15 points) Suppose A is a 3×4 matrix, B is a 4×5 matrix, and $AB = 0$. Prove that

$$\text{rank}(A) + \text{rank}(B) \leq 4.$$

5. (10 points) If the special solutions to $R\mathbf{x} = \mathbf{0}$ are in the columns of these N, go backward to find the nonzero rows of the reduced matrices R:

(a) (4 points) $N = \begin{bmatrix} 2 & 3 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$; (b) (3 points) $N = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$; (c) (3 points) $N = \begin{bmatrix} \\ \\ \end{bmatrix}$ (empty 3 by 1).

6. (15 points) Consider the closest cubic $b = C + Dt + Et^2 + Ft^3$ to the four points with $b = 0, 8, 8, 20$ at $t = 0, 1, 3, 4$. Write the four equations $A\mathbf{x} = \mathbf{b}$. Solve them by elimination. This cubic now goes exactly through the four points. What are $\mathbf{p}$ and $\mathbf{e}$?

7. (20 points) What is the closest function $a \cos x + b \sin x$ to the function $f(x) = \sin 2x$ on the interval from $-\pi$ to π? What is the closest straight line $c + dx$?