Linear and Nonlinear Computation Models
(CSE 4190.313)

Midterm Exam: April 20, 2016

Problem	Score
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Total	

Name: ____________________

ID No: ____________________

Dept: ____________________

1. (15 points) Construct a matrix A with entries $1, 2, \cdots, 9$. All rows and columns and diagonals add to 15. The first row could be 8, 3, 4. Show your work.

2. (15 points) Starting from a 3 by 3 matrix A with pivots 2, 7, 6, add a fourth row and column to produce M.

   (a) (3 points) What are the first three pivots for M?

   (b) (7 points) Explain why.

   (c) (5 points) What fourth row and column are sure to produce 9 as the fourth pivot?

3. (15 points) Given an $m \times n$ matrix A with rank r, if you know a particular solution $\mathbf{x}_p$ (free variables $= 0$) and all special solutions $\mathbf{x}_1, \cdots, \mathbf{x}_{n-r}$, for

$$A\mathbf{x} = \mathbf{b},$$

find a solution $\begin{bmatrix} \mathbf{y}_p \\ \mathbf{Y}_p \end{bmatrix}$ and all special solutions $\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{Y}_1 \end{bmatrix}, \cdots, \begin{bmatrix} \mathbf{y}_{2n-2r} \\ \mathbf{Y}_{2n-2r} \end{bmatrix}$, for

$$\begin{bmatrix} A & 2A \\ 3A & 7A \end{bmatrix} \begin{bmatrix} \mathbf{y} \\ \mathbf{Y} \end{bmatrix} = \begin{bmatrix} \mathbf{b} \\ 4\mathbf{b} \end{bmatrix}.$$

4. (15 points) The average of the four times t_i, $(i = 1, 2, 3, 4)$, is $\bar{t} = \frac{1}{4}(0 + 1 + 3 + 4) = 2$. Moreover, the average of the four b_i, $(i = 1, 2, 3, 4)$, is $\bar{b} = \frac{1}{4}(0 + 8 + 8 + 20) = 9$.

   (a) (7 points) Show that the best line goes through the center point $(\bar{t}, \bar{b}) = (2, 9)$.

   (b) (8 points) Explain why $C + D\bar{t} = \bar{b}$ comes from the first equation in $A^T A\, \hat{\mathbf{x}} = A^T \mathbf{b}$.

5. (15 points) Apply the Gram-Schmidt process to

$$\mathbf{a} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix},$$

and write the result in the form $A = QR$.

6. (15 points) What do you know about $C(A)$ when the number of solutions to $A\mathbf{x} = \mathbf{b}$ is

   (a) (5 points) 0 or 1, depending on $\mathbf{b}$.

   (b) (5 points) ∞, independent of $\mathbf{b}$.

   (c) (5 points) 0 or ∞, depending on $\mathbf{b}$.

7. (10 points)

   (a) (5 points) In the following Gram-Schmidt formula, show that C is orthogonal to $\mathbf{q}_1$ and $\mathbf{q}_2$:

$$C = \mathbf{c} - (\mathbf{q}_1^T\mathbf{c})\mathbf{q}_1 - (\mathbf{q}_2^T\mathbf{c})\mathbf{q}_2.$$

   (b) (5 points) In the following modified Gram-Schmidt steps, show that the vector $\bar{C}$ is the same as the vector C in the above equation:

$$C^* = \mathbf{c} - (\mathbf{q}_1^T\mathbf{c})\mathbf{q}_1, \quad \bar{C} = C^* - (\mathbf{q}_2^T C^*)\mathbf{q}_2.$$