Linear and Nonlinear Computation Models

(CSE 4190.313)

Midterm Exam: April 25, 2018

Problem	Score
1	
2	
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Total	

Name: ____________________

ID No: ____________________

Dept: ____________________

1. (15 points) Assuming all of A, B, $A+B$ are invertible, show that $C = B^{-1} + A^{-1}$ is also invertible, and find a formula for C^{-1}.

2. (15 points) Find a basis for the orthogonal complement of the row space of A:

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 4 \end{bmatrix}.$$

Split $\mathbf{x} = (3, 3, 3)^T$ into a row space component $\mathbf{x}_r$, and a nullspace component $\mathbf{x}_n$.

3. (10 points) Prove that the trace of $P = \mathbf{a}\mathbf{a}^T/\mathbf{a}^T\mathbf{a}$ always equals 1.

4. (15 points) Find the projection of $\mathbf{b}$ onto the column space of A:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ -2 & 4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 7 \end{bmatrix}.$$

5. (15 points)

   (a) (5 points) If the columns of A are orthogonal to each other, what can you say about the form of $A^T A$? If the columns are orthonormal, what can you say then?

   (b) (10 points) Under what conditions on the columns of A (which may be rectangular) is $A^T A$ invertible? Justify your answer.

6. (15 points) Find the fourth Legendre polynomial, which is orthogonal to 1, x, and $x^2 - \frac{1}{3}$, over the interval $-1 \leq x \leq 1$.

7. (15 points) The 4×4 *Hadamard matrix* H is given as follows:

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}.$$

Find H^{-1} and write $\mathbf{v} = (7, 5, 3, 1)^T$ as a combination of the columns of H.