Linear and Nonlinear Computation Models

(CSE 4190.313)

Midterm Exam: April 18, 2019

Problem	Score	Problem	Score
1		6	
2		7	
3		8	
4		9	
5		Total	

Name: _____

ID No: _____

Phone: _____

E-mail: _____

1. (10 points) Suppose P is the projection matrix onto the subspace S and Q is the projection onto the orthogonal complement $S^{\perp}$. What are $P+Q$ and PQ? Show that $P-Q$ is its own inverse.

2. (10 points) Show that the length of $A\mathbf{x}$ equals the length of $A^T\mathbf{x}$ if $AA^T = A^TA$.

3. (10 points) The lines $3x + y = b_1$ and $6x + 2y = b_2$ are _____. They are the same line

   if _____. In that case. (b_1, b_2) is perpendicular to the vector _____.

   The nullspace of the matrix is the line $3x + y =$ _____. One particular vector in

   that nullspace is _____.

4. (10 points) If you know the average $\hat{x}_N$ of N numbers $b_1, \cdots, b_N$, how can you quickly find the average $\hat{x}_{N+1}$ with one more number b_{N+1}? The idea of recursive least squares is to avoid adding $N + 1$ numbers.

5. (10 points) If $P_C = A(A^TA)^{-1}A^T$ is the projection onto the column space of A, what is the projection P_R onto the row space of A? Under what condition on A? Justify your answer.

6. (10 points) What matrix transforms $\begin{bmatrix} a \\ c \end{bmatrix}$ to $\begin{bmatrix} r \\ t \end{bmatrix}$ and $\begin{bmatrix} b \\ d \end{bmatrix}$ to $\begin{bmatrix} s \\ u \end{bmatrix}$?

7. (15 points) If $A\mathbf{x} = \mathbf{0}$ has a nonzero solution, show that $A^T\mathbf{y} = \mathbf{f}$ fails to be solvable for some right-hand sides $\mathbf{f}$. Construct an example of A and $\mathbf{f}$.

8. (15 points) Show that two matrices $A_{m\times n}$ and $B_{n\times r}$ can be multiplied as follows:

$$AB = \sum_{k=1}^{n} (\text{column } k \text{ of } A)(\text{row } k \text{ of } B) = \sum_{k=1}^{n} \mathbf{a}_k \mathbf{b}_k^T,$$

where $A = [\mathbf{a}_1 \cdots \mathbf{a}_n]$ and $B^T = [\mathbf{b}_1 \cdots \mathbf{b}_n]$.

9. (10 points) If an invertible matrix $A = L_1 D_1 U_1 = L_2 D_2 U_2$, show that the factorization is unique.