Quiz #2 (CSE 4190.313)

Monday, April 11, 2016

Name: ________________________ **ID No:** ________________________

1. (10 points) Given an $m \times n$ matrix A with rank r, if you know a particular solution $\mathbf{x}_p$ (free variables $= 0$) and all special solutions $\mathbf{x}_1, \cdots, \mathbf{x}_{n-r}$, for

$$A\mathbf{x} = \mathbf{b},$$

(a) (5 points) Find a solution $\begin{bmatrix} \mathbf{y}_p \\ \mathbf{Y}_p \end{bmatrix}$ and all special solutions $\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{Y}_1 \end{bmatrix}, \cdots, \begin{bmatrix} \mathbf{y}_{2n-r} \\ \mathbf{Y}_{2n-r} \end{bmatrix}$, for

$$\begin{bmatrix} A & 2A \\ 3A & 6A \end{bmatrix} \begin{bmatrix} \mathbf{y} \\ \mathbf{Y} \end{bmatrix} = \begin{bmatrix} \mathbf{b} \\ 3\mathbf{b} \end{bmatrix}.$$

(b) (5 points) Find a solution $\begin{bmatrix} \mathbf{y}_p \\ \mathcal{Y}_p \\ \mathbf{Y}_p \end{bmatrix}$ and all special solutions $\begin{bmatrix} \mathbf{y}_1 \\ \mathcal{Y}_1 \\ \mathbf{Y}_1 \end{bmatrix}, \cdots, \begin{bmatrix} \mathbf{y}_{3n-r} \\ \mathcal{Y}_{3n-r} \\ \mathbf{Y}_{3n-r} \end{bmatrix}$, for

$$\begin{bmatrix} A & A & A \end{bmatrix} \begin{bmatrix} \mathbf{y} \\ \mathcal{Y} \\ \mathbf{Y} \end{bmatrix} = \mathbf{b}.$$

2. (10 points)

   (a) (3 points) $A\mathbf{x} = \mathbf{b}$ has a solution under what conditions on $\mathbf{b}$, for the following A and $\mathbf{b}$?

$$A = \begin{bmatrix} 1 & 2 & 0 & 4 \\ 0 & 0 & 0 & 0 \\ 2 & 4 & 0 & 6 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

   (b) (2 points) Find a basis for the nullspace of A?

   (c) (3 points) Find the general solution to $A\mathbf{x} = \mathbf{b}$, when a solution exists.

   (d) (2 points) What is a basis for the column space of A?

3. (5 points) Suppose A is a symmetric matrix ($A^T = A$).

   (a) (2 points) Why is its column space perpendicular to its nullspace?

   (b) (3 points) If $A\mathbf{x} = \mathbf{0}$ and $A\mathbf{z} = 5\mathbf{z}$, which subspaces contain these eigenvectors $\mathbf{x}$ and $\mathbf{z}$? Explain why.

4. (5 points) What matrix P projects every point in $\mathbf{R}^3$ onto the line of intersection of the planes $x + y + t = 0$ and $x - t = 0$?

5. (6 points) (True or False) For the vector space M of all 3×3 matrices, justify your answer for the following problems:

   (a) (2 points) The skew-symmetric matrices in M (with $A^T = -A$) form a subspace.

   (b) (2 points) The unsymmetric matrices in M (with $A^T \neq A$) form a subspace.

   (c) (2 points) The matrices that have $(1, 1, 1)$ in their nullspace form a subspace.

6. (6 points) Apply elimination with an extra column to reach $R\mathbf{x} = \mathbf{0}$ and $R\mathbf{x} = \mathbf{d}$:

$$\begin{bmatrix} U & \mathbf{0} \end{bmatrix} = \begin{bmatrix} 3 & 3 & 6 & \mathbf{0} \\ 0 & 0 & 2 & \mathbf{0} \\ 0 & 0 & 0 & \mathbf{0} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} U & \mathbf{c} \end{bmatrix} = \begin{bmatrix} 3 & 3 & 6 & \mathbf{9} \\ 0 & 0 & 2 & \mathbf{4} \\ 0 & 0 & 0 & \mathbf{0} \end{bmatrix}$$

Solve $R\mathbf{x} = \mathbf{0}$ (free variable $= 1$). What are the solutions to $R\mathbf{x} = \mathbf{d}$?

7. (10 points) Construct a matrix with the required property, or explain why you can't.

(a) (2 points) Column space contains $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$; and row space contains $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$.

(b) (2 points) Column space has basis $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$; and nullspace has basis $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$.

(c) (2 points) Dimension of nullspace $= 1 +$ dimension of left nullspace.

(d) (2 points) Left nullspace contains $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$; and row space contains $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$.

(e) (2 points) Row space $=$ column space, and nullspace $\neq$ left nullspace.