Quiz #2 (CSE 4190.313)

Wednesday, April 11, 2018

Name: _____ **ID No:** _____

1. (10 points) A is an $m \times n$ matrix of rank r. Suppose there are right-hand sides $\mathbf{b}$ for which $A\mathbf{x} = \mathbf{b}$ has no solution.

   (a) (5 points) What inequalityes ($<$ or $\leq$) must be true between m, n, r? Explain why.

   (b) (5 points) How do you know that $A^T\mathbf{y} = \mathbf{0}$ has a nonzero solution?

2. (10 points) Under what condition on b_1, b_2, b_3 is the following system solvable? Find all solutions when that condition holds.

$$\begin{bmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 4 & 8 \\ 0 & 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

3. (10 points) Using the fact that the total number of 5×5 permutation matrices is 5!, answer the following yes/no questions.

   (a) (5 points) Are they linearly independent? Explain why.

   (b) (5 points) Do they span the space of all 5×5 matrices? Explain why.

4. (10 points) On the vector space $\mathbf{P}_3$ of cubic polynomials, what matrix represents $\frac{d^2}{dt^2}$? Construct the 4×4 matrix A from the standard basis $1, t, t^2, t^3$. Find its nullspace and column space. What do they mean in terms of polynomials?

5. (10 points) Find all vectors that are perpendicular to $(1, 4, 4, 1)$ and $(2, 9, 8, 2)$.