Quiz #4 (CSE 4190.313)

Monday, June 11, 2018

Name: ______________________ **ID No:** ______________________

1. (10 points) If $A = Q\Lambda Q^T$ is symmetric positive definite, then $R = Q\sqrt{\Lambda}\,Q^T$ is its symmetric positive definite square root. Compute R and verify $R^2 = A$ for the following matrices:

$$A_1 = \begin{bmatrix} 10 & 6 \\ 6 & 10 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 10 & -6 \\ -6 & 10 \end{bmatrix}.$$

2. (10 points) Compute the singular value decomposition (SVD) of the following matrix

$$A = \begin{bmatrix} 3 & 6 & 3 \\ 4 & 8 & 4 \end{bmatrix}.$$

3. (10 points) Show that $A^T A$ is symmetric positive definite when A has independent columns.

4. (5 points) Change $A\mathbf{x} = \mathbf{b}$ to $\mathbf{x} = (I - A)\mathbf{x} + \mathbf{b}$. Explain why the iteration

$$\mathbf{x}_{k+1} = (I - A)\mathbf{x}_k + \mathbf{b}$$

does not converge for $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$.

5. (15 points) For the matrix $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and the initial guess $\mathbf{u}_0 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$,

(a) (5 points) Apply three inverse power steps:

$$\mathbf{u}_{k+1} = A^{-1}\mathbf{u}_k = \frac{1}{3}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\mathbf{u}_k, \quad \text{for } k = 0, 1, 2,$$

(b) (7 points) And then apply just one shifted step $\mathbf{u} = (A - \alpha I)^{-1}\mathbf{u}_0$, with $\alpha = \frac{\mathbf{u}_0^T A \mathbf{u}_0}{\mathbf{u}_0^T \mathbf{u}_0}$.

(c) (3 points) Compare the results using the fact that $\mathbf{u}_k$ converges to a multiple of $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.