Quiz #3 (CSE 4190.313)

Tuesday, May 7, 2019

Name: ______________________ **ID No:** ______________________

1. (15 points) Apply the Gram-Schmidt process to

$$\mathbf{a} = \sin t, \quad \mathbf{b} = \cos t, \quad \mathbf{c} = 1,$$

under the inner product $\langle f, g \rangle = \int_0^\pi f(t)g(t)dt$.

2. (15 points)

   (a) (3 points) Find a nonzero matrix A such that A^3 is a zero matrix..

   (b) (2 points) If $A\mathbf{x} = \lambda\mathbf{x}$, for some $\mathbf{x} \neq \mathbf{0}$, show that λ must be zero.

   (c) (10 points) Prove that A cannot be symmetric.

Quiz #3-2 (CSE 4190.313)

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3. (5 points) If $A^2 = A$, show that $e^{At} = I + (e^t - 1)A$.

4. (5 points) Show that A_i and B_i $(i = 1, 2)$ are similar:

(a) $A_1 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$, (b) $A_2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $B_2 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$.

5. (10 points) Given two different bases $\{\mathbf{v}_1, \cdots, \mathbf{v}_n\}$, and $V_1, \cdots, V_n\}$, three $n \times n$ matrices $A = [a_{ij}]$, $B = [b_{ij}]$, $M = [m_{ij}]$ are defined as follows (for $j = 1, \cdots, n$):

$$\begin{aligned} T\mathbf{v}_j &= a_{1j}\mathbf{v}_1 + \cdots + a_{nj}\mathbf{v}_n, \\ TV_j &= b_{1j}V_1 + \cdots + ab_{nj}V_n, \\ V_j &= m_{1j}\mathbf{v}_1 + \cdots + m_{nj}\mathbf{v}_n, \end{aligned}$$

Show that $MB = AM$.