Quiz #3 (CSE 400.001)

Thursday, April 12, 2001

1. (7 points) Find the inverse Laplace transform of the following function

$$\ln \frac{s^2+1}{(s-1)^2}$$

$$F(s) = \ln \frac{s^2+1}{(s-1)^2}$$

$$F'(s) = 2\left[\frac{s}{s^2+1} - \frac{1}{s-1}\right]$$

(+2)

$$\mathcal{L}^{-1}\left[F'(s)\right] = 2\left(\cos t - e^{t}\right)$$

(+2)

$$= -t f(t)$$

$$\therefore f(t) = \frac{2}{t}\left(e^{t} - \cos t\right)$$

(+3)

2. (3 points) Find the Laplace transform of the following function

$$y(t) = \begin{cases} 1 - e^{-t} & \text{if } 0 < t < 2 \\ 0 & \text{otherwise} \end{cases}$$

$$y(t) = (1-e^{-t})\left(u(t) - u(t-2)\right)$$

$$= (1-e^{-t}) - \left(1 - e^{-(t-2)-2}\right) u(t-2)$$

(+1)

$$Y(s) = \frac{1}{s} - \frac{1}{s+1} - e^{-2s}\left[\frac{1}{s} - e^{-2}\cdot\frac{1}{s+1}\right]$$

(+2)

3. (5 points) Solve the following integral equation

$$y(t) = te^{t} - 2e^{t}\int_0^t e^{-\tau} y(\tau)\,d\tau$$

$$y(t) = te^{t} - 2\int_0^t e^{(t-\tau)} y(\tau)\,d\tau$$

(+2)

$$Y(s) = \frac{1}{(s-1)^2} - 2\cdot\frac{1}{s-1}\cdot Y(s)$$

(+1)

$$\frac{s+1}{s-1} Y(s) = \frac{1}{(s-1)^2}$$

$$Y(s) = \frac{1}{2}\left[\frac{1}{s-1} - \frac{1}{s+1}\right]$$

(+1)

$$y(t) = \frac{1}{2}\left[e^{t} - e^{-t}\right]$$

$$= \sinh t$$

(+1)