Quiz #2 (CSE 400.001)

Tuesday, March 25, 2003

Name: ____________________ E-mail: ____________________

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1. (10 points) Solve the following initial value problem:

$$y'' + y = 4x + 10\sin x, \quad y(\pi) = 0, \; y'(\pi) = 2.$$

$\lambda^2 + 1 = 0, \quad \lambda = \pm i$

$y_h = A\cos x + B\sin x$ (+1)

$y_p = k_1 x + k_0 + \boxed{Kx\cos x + Mx\sin x}$ (+2)

$y_p' = k_1 + K\cos x - Kx\sin x + M\sin x + Mx\cos x$

$y_p'' = -2K\sin x - Kx\cos x + 2M\cos x - Mx\sin x$ (+1)

$y_p'' + y_p = k_1 x + k_0 - 2K\sin x + 2M\cos x = 4x + 10\sin x$

$\therefore k_1 = 4, \; k_0 = 0, \; K = -5, \; M = 0$ (+2)

$y_p = 4x - 5x\cos x$

$y = \underline{A\cos x + B\sin x + 4x - 5x\cos x}$ (+1)

$y(\pi) = -A + 4\pi + 5\pi = 0 \Rightarrow A = 9\pi$

$y'(\pi) = -B + 4 + 5 = 2 \Rightarrow B = 7$ (+2)

$\therefore y = \underline{9\pi\cos x + 7\sin x + 4x - 5x\cos x}$ (+1)

2. (15 points) Using the substitution $x = e^t$, solve the following differential equation:

$$x^3y''' - 3x^2y'' + 6xy' - 6y = 3 + \ln x^3.$$

+1 $\underline{\hat{y}(t) = y(e^t)}$, $\underline{\hat{y}'(t) = y'(e^t)\cdot e^t}$ +1

$\hat{y}''(t) = y''(e^t)\cdot e^{2t} + y'(e^t)\cdot e^t = y''(e^t)\cdot e^{2t} + \hat{y}'(t)$ +1

$\hat{y}'''(t) = y'''(e^t)\cdot e^{3t} + 2y''(e^t)\cdot e^{2t} + \hat{y}''(t)$

$= y'''(e^t)\cdot e^{3t} + 3\hat{y}''(t) - 2\hat{y}'(t)$ +2

$[\hat{y}'''(t) - 3\hat{y}''(t) + 2\hat{y}'(t)]$

$-3[\hat{y}''(t) - \hat{y}'(t)] + 6\hat{y}'(t) - 6\hat{y}(t) = 3 + 3t$ +4

$\hat{y}'''(t) - 6\hat{y}''(t) + 11\hat{y}'(t) - 6\hat{y}(t) = 3 + 3t$ +1

$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0 \Rightarrow (\lambda-1)(\lambda-2)(\lambda-3) = 0$

$\hat{y}_h(t) = c_1e^t + c_2e^{2t} + c_3e^{3t}$ +1

+1 $\underline{\hat{y}_p(t) = k_1t + k_0}$ $\Rightarrow \hat{y}_p'(t) = k_1,\ \hat{y}_p''(t) = \hat{y}_p'''(t) = 0$

$11k_1 - 6(k_1t + k_0) = 3 + 3t \Rightarrow k_1 = -\frac{1}{2},\ k_0 = -\frac{17}{12}$

$\therefore \hat{y}_p(t) = -\frac{1}{2}t - \frac{17}{12}$ +1

$\hat{y}(t) = c_1e^t + c_2e^{2t} + c_3e^{3t} - \frac{1}{2}t - \frac{17}{12}$ +1

$y(x) = c_1x + c_2x^2 + c_3x^3 - \frac{1}{2}\ln x - \frac{17}{12}$ +1