Quiz #4 (CSE 400.001)

Wednesday, November 9, 2011

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1. (10 points) Find the Laplace transform of the following function:

$$t \int_0^t \tau \sin \tau d\tau$$

$$\mathcal{L}[\sin t] = \frac{1}{s^2+1} \quad +2$$

$$\mathcal{L}[t \sin t] = -\frac{d}{ds}\left[\frac{1}{s^2+1}\right] = \frac{2s}{(s^2+1)^2} \quad +3$$

$$\mathcal{L}\left[\int_0^t \tau \sin \tau d\tau\right] = \frac{1}{s} \cdot \frac{2s}{(s^2+1)^2} = \frac{2}{(s^2+1)^2} \quad +3$$

$$\mathcal{L}\left[t \int_0^t \tau \sin \tau d\tau\right] = -\frac{d}{ds}\left[\frac{2}{(s^2+1)^2}\right]$$

$$= \frac{8s}{(s^2+1)^3} \quad +2$$

2. (10 points) Solve the following integral equation

$$f(t) = 3t^2 - te^{-t} - e^t \int_0^t e^{-\tau} f(\tau) d\tau$$

$$f(t) = 3t^2 - te^{-t} - \int_0^t e^{t-\tau} f(\tau) d\tau$$

$$= 3t^2 - te^{-t} - e^t * f(t) \quad +3$$

$$F(s) = 3 \cdot \frac{2!}{s^3} - \frac{1}{(s+1)^2} - \frac{1}{s-1} \cdot F(s) \quad +2$$

$$\frac{s}{s-1} \cdot F(s) = \frac{6}{s^3} - \frac{1}{(s+1)^2}$$

$$F(s) = \frac{6(s-1)}{s^4} - \frac{s-1}{s(s+1)^2}$$

$$= 3 \cdot \frac{2!}{s^3} - \frac{3!}{s^4} \quad +3$$

$$+ \left[+ \frac{1}{s} - \frac{1}{s+1} - \frac{2}{(s+1)^2} \right]$$

$$f(t) = 3 \cdot t^2 - t^3 + 1 - e^{-t} - 2te^{-t}$$

+2