

Quiz #7 (CSE 400.001)

November 30, 2011 (Wednesday)

Name: _____ E-mail: _____

Dept: _____ ID No: _____

1. (10 points) Diagonalize A and compute $S\Lambda^k S^{-1}$

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\det(A - \lambda I) = \begin{vmatrix} 2 - \lambda & 1 \\ 0 & 1 - \lambda \end{vmatrix} = (\lambda - 2)(\lambda - 1) = 0$$

$$\lambda_1 = 2, \quad \mathbf{x}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}; \quad \lambda_2 = 1, \quad \mathbf{x}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$S = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}, \quad S^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}, \quad \Lambda = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A = S\Lambda S^{-1}$$

$$\begin{aligned} A^k &= S\Lambda^k S^{-1} \\ &= \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2^k & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 2^k & 2^k - 1 \\ 0 & 1 \end{bmatrix} \end{aligned}$$