Quiz #1 (CSE 400.001)

Monday, September 15, 2014

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1. (~~7~~ 8 points) Solve the following initial value problem:

$$\frac{dy}{dx} = \frac{xy^2 - \cos x \sin x}{y(1-x^2)}, \quad y(0) = 2.$$

$$(xy^2 - \cos x \sin x)\,dx + y(x^2-1)\,dy = 0 \quad (+1)$$

$$\frac{\partial M}{\partial y} = 2xy = \frac{\partial N}{\partial x} : \text{exact} \quad (+1)$$

$$u = \int y(x^2-1)\,dy + l(x) = \frac{1}{2}y^2(x^2-1) + l(x) \quad (+1)$$

$$\frac{\partial u}{\partial x} = xy^2 + l'(x) = M = xy^2 - \frac{1}{2}\sin 2x \quad (+1)$$

$$\therefore l'(x) = -\frac{1}{2}\sin 2x, \quad l(x) = \frac{1}{4}\cos 2x + C \quad (+1)$$

$$u(x,y) = \frac{1}{2}y^2(x^2-1) + \frac{1}{4}\cos 2x + C = 0$$

$$u(0,2) = \frac{1}{2}\cdot 4(-1) + \frac{1}{4} + C = 0 \Rightarrow C = \frac{7}{4} \quad (+2)$$

$$\therefore u(x,y) = \frac{1}{2}y^2(x^2-1) + \frac{1}{4}\cos 2x + \frac{7}{4} = 0 \quad (+1)$$

or $2y^2(x^2-1) + \cos 2x + 7 = 0$

2. 7 (~~8~~ points) Solve the following initial value problem:

$$(x+2)^2 \frac{dy}{dx} = 5 - 8y - 4xy, \quad y(0) = 1.$$

$$y' + \frac{4}{x+2} y = \frac{5}{(x+2)^2}$$ +1

$$h(x) = \int \frac{4}{x+2}\,dx = 4\ln|x+2|$$ +1

$$y = e^{-h(x)}\left[\int e^{h(x)} \cdot r(x)\,dx + c\right]$$

$$= \frac{1}{(x+2)^4}\left[\int (x+2)^4 \cdot \frac{5}{(x+2)^2}\,dx + c\right]$$

$$= \frac{1}{(x+2)^4} \cdot \left[\int 5(x+2)^2\,dx + c\right]$$ +3

$$= \frac{1}{(x+2)^4}\left[\frac{5}{3}(x+2)^3 + c\right]$$

$$= \frac{c}{(x+2)^4} + \frac{5}{3} \cdot \frac{1}{(x+2)}$$

$$1 = \frac{c}{2^4} + \frac{5}{6} \Rightarrow c = \frac{1}{6} \cdot 2^4 = \frac{8}{3}$$ +1

$$\therefore y = \frac{5}{3} \cdot \frac{1}{(x+2)} + \frac{8}{3} \cdot \frac{1}{(x+2)^4}, \quad x > -2$$

+1