Quiz #2 (CSE 400.001)

Monday, September 29, 2014

Name: ____________________ E-mail: ____________________

Dept: ____________________ ID No: ____________________

1. (8 points) Find a formula involving integrals for a particular solution of the following differential equation:

$$y''' - y'' - y' + y = f(x).$$

$$\lambda^3 - \lambda^2 - \lambda + 1 = (\lambda-1)(\lambda^2-1) = (\lambda-1)^2(\lambda+1) = 0$$

$$y_1 = e^x, \quad y_2 = xe^x, \quad y_3 = e^{-x} \qquad +2$$

$$W = \begin{vmatrix} e^x & xe^x & e^{-x} \\ e^x & (1+x)e^x & -e^{-x} \\ e^x & (2+x)e^x & e^{-x} \end{vmatrix} = e^x \begin{vmatrix} 1 & x & 1 \\ 1 & 1+x & -1 \\ 1 & 2+x & 1 \end{vmatrix} = 4e^x \qquad +1$$

$$W_1 = \begin{vmatrix} 0 & x & 1 \\ 0 & 1+x & -1 \\ 1 & 2+x & 1 \end{vmatrix} = -2x-1$$

$$W_2 = 2, \qquad W_3 = e^{2x} \qquad +3$$

$$y_p = e^x \int \frac{-2x-1}{4e^x} f(x)\,dx + xe^x \int \frac{2}{4e^x} f(x)\,dx + e^{-x} \int \frac{e^{2x}}{4e^x} f(x)\,dx \qquad +2$$

2. (12 points) Solve the following initial value problem:

$$x^3y''' - 3x^2y'' + 6xy' - 6y = x^4, \ (x > 0), \quad y(1) = 0, y'(1) = 0, y''(1) = 0.$$

$$m(m-1)(m-2) - 3m(m-1) + 6m - 6$$
$$= (m-1)(m-2)(m-3) = 0 \qquad +1$$

$$y_1 = x, \quad y_2 = x^2, \quad y_3 = x^3 \qquad +1$$

Let $y_p = cx^4$ $\quad +1$

$$y_p' = 4cx^3, \quad y_p'' = 12cx^2, \quad y_p''' = 24cx$$

$$(24 - 36 + 24 - 6)c \cdot x^4 = x^4$$

$$\therefore c = \frac{1}{6}, \quad y_p = \frac{1}{6}x^4 \qquad +2$$

$$y = y_h + y_p = c_1x + c_2x^2 + c_3x^3 + \frac{1}{6}x^4 \qquad +2$$

$$y' = c_1 + 2c_2x + 3c_3x^2 + \frac{2}{3}x^3$$
$$y'' = 2c_2 + 6c_3x + 2x^2 \qquad +1$$

$$y(1) = c_1 + c_2 + c_3 + \frac{1}{6} = 0$$
$$y'(1) = c_1 + 2c_2 + 3c_3 + \frac{2}{3} = 0$$
$$y''(1) = 2c_2 + 6c_3 + 2 = 0$$
$$\therefore c_1 = -\frac{1}{6}, \quad c_2 = \frac{1}{2}, \quad c_3 = -\frac{1}{2} \qquad +3$$

$$y = -\frac{1}{6}x + \frac{1}{2}x^2 - \frac{1}{2}x^3 + \frac{1}{6}x^4 \qquad +1$$