Quiz #4 (CSE 400.001)

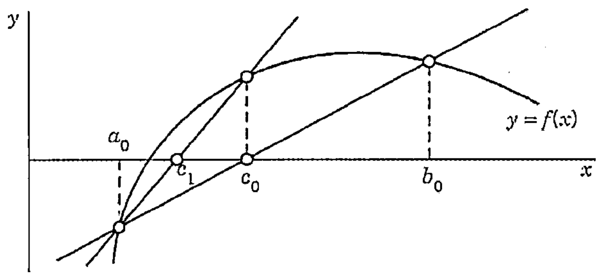
Monday, November 24, 2014

Name: ______________________ **E-mail:** ______________________

Dept: ______________________ **ID No:** ______________________

1. (10 points) The following figure shows the regula falsi method. Starting with $x_0 = 0$ and $x_1 = 2$, show the first two steps of the regula falsi method (i.e., compute x_2 and x_3) in solving the following equation:

$$x^2 - 4x + 3 = 0.$$



$x_0 = 0,\ y_0 = 3\ ;\ x_1 = 2,\ y_1 = -1$

$$0 = \frac{y_1 - y_0}{x_1 - x_0}(x_2 - x_0) + y_0$$

$$x_2 = \frac{(x_0 - x_1)\,y_0}{y_1 - y_0} + x_0 = \frac{x_0 y_1 - x_1 y_0}{y_1 - y_0}$$

$$\therefore x_2 = \frac{-6}{-4} = \frac{3}{2},\quad y_2 = \left(\frac{3}{2}\right)^2 - 4\cdot\frac{3}{2} + 3 = -\frac{3}{4}$$

$$x_3 = \frac{x_0 y_2 - x_2 y_0}{y_2 - y_0} = \frac{-\frac{3}{2}\cdot 3}{-\frac{3}{4} - 3} = \frac{9/2}{15/4} = \frac{6}{5}$$

$$\therefore x_2 = \frac{3}{2},\quad x_3 = \frac{6}{5}$$

2. (10 points) Assuming that

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi}{L} x + b_n \sin \frac{n\pi}{L} x \right),$$

show that Parseval's identity holds:

$$\frac{1}{L} \int_{-L}^{L} [f(x)]^2 \, dx = 2a_0^2 + \sum_{n=1}^{\infty} (a_n^2 + b_n^2).$$

$$\int_{-L}^{L} [f(x)]^2 dx$$

$$= \int_{-L}^{L} a_0^2 dx + 2a_0 \sum_{n=1}^{\infty} \left(a_n \cancelto{0}{\int_{-L}^{L} \cos \frac{n\pi}{L} x \, dx} + b_m \cancelto{0}{\int_{-L}^{L} \sin \frac{n\pi}{L} x \, dx} \right)$$

$$+ \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(a_m a_n \boxed{\int_{-L}^{L} \cos \frac{m\pi}{L} x \cos \frac{n\pi}{L} x \, dx} = \begin{cases} 0 & \text{if } m \neq n \\ L & \text{if } m = n \end{cases} \right.$$

$$+ a_m b_n \cancelto{0}{\int_{-L}^{L} \cos \frac{m\pi}{L} x \sin \frac{n\pi}{L} x \, dx}$$

$$+ a_n b_m \cancelto{0}{\int_{-L}^{L} \cos \frac{n\pi}{L} x \sin \frac{m\pi}{L} x \, dx}$$

$$+ b_m b_n \boxed{\int_{-L}^{L} \sin \frac{m\pi}{L} x \sin \frac{n\pi}{L} x \, dx} = \begin{cases} 0 & \text{if } m \neq n \\ L & \text{if } m = n \end{cases}$$

$$= 2La_0^2 + \sum_{n=1}^{\infty} \left(a_n^2 \int_{-L}^{L} \cos^2 \frac{n\pi}{L} x \, dx + b_m^2 \int_{-L}^{L} \sin^2 \frac{n\pi}{L} x \, dx \right)$$

$$= 2La_0^2 + L \sum_{n=1}^{\infty} (a_n^2 + b_n^2)$$

$$\therefore \frac{1}{L} \int_{-L}^{L} [f(x)]^2 dx = 2a_0^2 + \sum_{n=1}^{\infty} (a_n^2 + b_m^2)$$

3. (15 points) Table 1 shows the result of applying the Runge-Kutta method to the following initial value problem

$$xy' = x - y, \quad y(2) = 2,$$

from $x = 2$ to $x = 3$ with $h = 0.2$. Fill in the blank and show your work for partial credit.

x_i	y_i
2.0	2.0000
2.2	2.0091
2.4	
2.6	2.0692
2.8	2.1143
3.0	2.1667

Table 1: Runge-Kutta Method

$$f(x,y) = y' = \frac{x-y}{x}$$

$$k_1 = hf(x_1, y_1) = 0.2 * \frac{2.2 - 2.0091}{2.2} \approx 0.0174$$

$$k_2 = hf(x_1 + \tfrac{1}{2}h, y_1 + \tfrac{1}{2}k_1)$$

$$= 0.2 * \frac{2.3 - (2.0091 + 0.0087)}{2.3} \approx 0.0245$$

$$k_3 = 0.2 * \frac{2.3 - (2.0091 + 0.0123)}{2.3} \approx 0.0242$$

$$k_4 = hf(x_1 + h, y_1 + k_3)$$

$$= 0.2 * \frac{2.4 - (2.0091 + 0.0242)}{2.4} \approx 0.0306$$

$$y_2 = y_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$\approx 2.0333$$