Quiz #1 (EngMath I) [Monday, Sept. 14, 2015]

Name: _____ ID No: _____

1. (7 points) Solve the following initial value problem:

$$\left(\frac{1}{1+y^2}+\cos x-2xy\right)\frac{dy}{dx}=y(y+\sin x),\quad y(0)=1.$$

$y(y+\sin x)dx+\left(2xy-\cos x-\frac{1}{1+y^2}\right)dy=0$ (+1)

$\frac{\partial M}{\partial y}=2y+\sin x=\frac{\partial N}{\partial x}$: exact (+1)

$u(x,y)=\int M(x,y)\,dx+k(y)$ (+1)

$=y^2x-y\cos x+k(y)$

$N=2xy-\cos x+k'(y)=2xy-\cos x-\frac{1}{1+y^2}$ (+1)

$\therefore k'(y)=-\frac{1}{1+y^2}$ (+1)

$k(y)=-\arctan(y)+C^*$ (+1)

$u(x,y)=xy^2-y\cos x-\arctan(y)=C$

$u(0,1)=-1-\arctan(1)=C$ (+1)

$\therefore u(x,y)=xy^2-y\cos x-\arctan(y)+1+\frac{\pi}{4}=0$

2. (8 points) Solve the following Bernoulli equation:

$$x\frac{dy}{dx} + y = x^2y^2.$$

$y' + \frac{1}{x}y = xy^2$ (+1)

$u = y^{-1}$ (+1)

$u' = -\frac{y'}{y^2}$ (+1)

$-\frac{1}{u^2}u' + \frac{1}{x}\cdot\frac{1}{u} = x\cdot\frac{1}{u^2}$ (+1)

$u' - \frac{1}{x}u = -x$ (+1)

$u = e^{\int\frac{1}{x}dx}\left[\int e^{-\int\frac{1}{x}dx}\cdot(-x)dx + C\right]$ (+1)

$= x\left[-x + C\right]$] (+1)

$= -x^2 + Cx$

$\therefore y = \frac{1}{u} = \frac{1}{-x^2 + Cx}$ (+1)