

Quiz #4 (EngMath I) [Monday, Nov. 16, 2015]

Name: _____ ID No: _____

1. (10 points) Find the cubic spline $g(x)$ to the following data, with $k_0 = 0$ and $k_3 = -6$:

$$f_0 = f(0) = 1, f_1 = f(1) = 0, f_2 = f(2) = -1, f_3 = f(3) = 0.$$

2. (15 points) Consider the following hyperbolic equation

$$u_{tt} = u_{xx} + 100, \quad 0 \leq x \leq 1, \quad 0 \leq t \leq 0.4,$$

with initial and boundary conditions

$$u(x, 0) = x^3, \quad u_t(x, 0) = x^2; \quad u_x(0, t) = t^2, \quad u(1, t) = (1 + t)^3,$$

Approximate the solution to the above equation with $h = k = 0.2$, for $0 \leq t \leq 0.4$.

(a) (5 points) Represent $u_{i,j+1}$ in terms of $u_{i-1,j}, u_{i,j}, u_{i+1,j}, u_{i,j-1}$.

(b) (5 points) Represent $u_{i,1}$ in terms of $u_{i-1,0}, u_{i,0}, u_{i+1,0}$.

(c) (5 points) Represent $u_{0,j+1}$ in terms of $u_{0,j}, u_{1,j}, u_{0,j-1}$.