

Quiz #5 (EngMath I) [Monday, Nov. 30, 2015]

Name: \_\_\_\_\_ ID No: \_\_\_\_\_

1. (10 points) What is the operation count for the Gauss-Jordan method in solving  $n$  systems

$$A\mathbf{x}_i = \mathbf{e}_i, \quad \text{for } i = 1, \dots, n,$$

where  $\mathbf{e}_i$  is the  $i$ -th column of the identity matrix  $I$ .

- (a) (2 points) How many steps does the  $LU$ -factorization  $A = LU$  take?
- (b) (3 points) How many steps does each forward elimination  $L\mathbf{c}_i = \mathbf{e}_i$  take?
- (c) (2 points) How many steps does each back-substitution  $U\mathbf{x}_i = \mathbf{c}_i$  take?
- (d) (3 points) How many steps does the overall elimination take?

2. (5 points) Given two  $n \times n$  matrices  $A$  and  $B$ , show that  $(AB)_{ij}^T = (B^T A^T)_{ij}$ , for  $i, j = 1, \dots, n$ .

3. (5 points) Assuming  $a_{11} \neq 0$  and  $a_{22} \neq 0$ , find a nonzero solution  $\mathbf{x} = (x_1, x_2, x_3, x_4)^T \neq \mathbf{0}$  for the following matrix equation:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & 0 & a_{34} \\ 0 & 0 & 0 & a_{44} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$