Quiz #1 (CSE 400.001)

Tuesday, March 19, 2002

Name: ______________________ E-mail: ______________________

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1. (5 points) Find the general solution of the following differential equation.

$$\frac{y^2}{2} + 2ye^x + (y + e^x)\frac{dy}{dx} = 0$$

$$\underbrace{\left(\frac{y^2}{2} + 2ye^x\right)}_{=P} dx + \underbrace{(y + e^x)}_{=Q} dy = 0 \quad +1$$

$$\frac{1}{Q}\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = \frac{1}{y+e^x}(y + 2e^x - e^x) = 1 \quad +1$$

$$F(x) = \exp\left(\int 1\, dx\right) = e^x \quad +1$$

$$\left(\frac{1}{2}y^2e^x + 2ye^{2x}\right)dx + (ye^x + e^{2x})dy = 0 \quad : \text{exact}$$

$$u(x,y) = \frac{1}{2}y^2e^x + ye^{2x} + k(y) \quad +1$$

$$\frac{\partial u}{\partial y} = ye^x + e^{2x} + k'(y) = ye^x + e^{2x}$$

$$\therefore k(y) = \text{const}$$

$$\therefore u(x,y) = \frac{1}{2}y^2e^x + ye^{2x} + c = 0 \quad +1$$

2. (4 points) Solve the following initial value problem

$$xy' + 4y = 8x^4, \quad y(1) = 2.$$

$$y' + \frac{4}{x}y = 8x^3 \quad (+1)$$

$$y = e^{-\int \frac{4}{x}dx} \cdot \left[\int e^{\int \frac{4}{x}dx} \cdot 8x^3\,dx + c \right]$$

$$= e^{-4\ln x} \cdot \left[\int e^{4\ln x} \cdot 8x^3\,dx + c \right]$$

$$= x^{-4} \cdot \left[\int 8 \cdot x^7\,dx + c \right]$$

$$= x^{-4}\left[x^8 + c \right]$$

$$= x^4 + c \cdot x^{-4}$$

(+2)

$$2 = 1 + c \qquad \therefore c = 1$$

$$\therefore y = x^4 + x^{-4}$$

(+1)

3. (6 points) Apply Picard's iteration to the following problem. Compute $y_1(x)$ and $y_2(x)$.

$$y' = \frac{3y}{x}, \quad y(1) = 1.$$

$$f(x,y) = \frac{3y}{x}, \quad x_0 = 1, \quad y_0 = 1 \quad (+1)$$

$$y_1 = y_0 + \int_1^x f(t, y_0)\,dt = 1 + \int_1^x \frac{3}{t}\,dt = 1 + 3\ln x \quad (+2)$$

$$y_2 = y_0 + \int_1^x f(t, y_1)\,dt = 1 + \int_1^x \frac{3 + 9\ln t}{t}\,dt$$

$$= 1 + 3\ln x + 9 \cdot \int_1^x \frac{\ln t}{t}\,dt$$

$$= 1 + 3\ln x + 9 \cdot \left[\frac{1}{2}(\ln t)^2 \right]_1^x$$

$$= 1 + 3\ln x + \frac{9}{2}(\ln x)^2 \quad (+3)$$