Quiz #1 (CSE 400.001)

Thursday, March 13, 2003

Name: ________________ **E-mail:** ________________

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1. (6 points) Solve the following initial-value problem:

$$y' = (-2x + y)^2 - 7, \quad y(0) = 0.$$

$u = -2x + y, \quad u' = -2 + y', \quad y' = u' + 2$ (+1)

$u' + 2 = u^2 - 7, \quad u' = u^2 - 9$ (+1)

$$\frac{du}{u^2 - 9} = dx \quad (+1)$$

$$\frac{1}{6}\left[\frac{1}{u-3} - \frac{1}{u+3}\right] du = dx \quad (+1)$$

$$\frac{1}{6} \ln\left|\frac{u-3}{u+3}\right| = x + c_1$$

$$\frac{u-3}{u+3} = \pm e^{6x} \cdot e^{6c_1} = c \cdot e^{6x}$$

$$u = 3 \cdot \frac{1 + ce^{6x}}{1 - ce^{6x}}$$

(+1)

$$y = 2x + 3\left(\frac{1 + ce^{6x}}{1 - ce^{6x}}\right)$$

$$0 = 0 + 3 \cdot \frac{1+c}{1-c}$$

$$\therefore c = -1$$

$$y = 2x + 3 \cdot \frac{1 - e^{6x}}{1 + e^{6x}}$$

(+1)

2. (4 points) Find the general solution of the following differential equation.

$$xy' + y = x^2y^2.$$

$$u = y^{1-2} = y^{-1} = \frac{1}{y} \quad (+1)$$

$$u' = -\frac{1}{y^2}\cdot y'$$

$$y' = -y^2\cdot u' = -\frac{1}{u^2}\cdot u'$$

$$x\cdot\left(-\frac{1}{u^2}\cdot u'\right) + \frac{1}{u} = x^2\cdot\frac{1}{u^2} \quad (+1)$$

$$u' - \frac{1}{x}u = -x$$

$$u = e^{-\int(-\frac{1}{x})dx}\left[\int e^{\int(-\frac{1}{x})dx}\cdot(-x)\,dx + c\right]$$

$$= x\cdot\left[\int(-1)\,dx + c\right] = cx - x^2 \quad (+1)$$

$$y = \frac{1}{u} = \frac{1}{cx - x^2} \quad (+1)$$

3. (5 points) Apply Picard's iteration to the following problem. Compute $y_1(x)$ and $y_2(x)$.

$$x^3y' + 3x^2y = \frac{1}{x}, \quad y(1) = 0.$$

$$y' = f(x,y) = -\frac{3}{x}y + \frac{1}{x^4} = -3x^{-1}y + x^{-4}, \quad x_0 = 1,\ y_0 = 0 \quad (+2)$$

$$y_1 = y_0 + \int_{x_0}^{x}(-3t^{-1}y_0 + t^{-4})\,dt$$

$$= \int_1^x t^{-4}\,dt = \left[-\frac{1}{3}t^{-3}\right]_1^x = \frac{1}{3}(1 - x^{-3}) \quad (+1)$$

$$y_2 = y_0 + \int_{x_0}^{x}(-3t^{-1}y_1 + t^{-4})\,dt$$

$$= \int_1^x (2t^{-4} - t^{-1})\,dt = \left[-\frac{2}{3}t^{-3} - \ln t\right]_1^x \quad (+2)$$

$$= \frac{2}{3}(1 - x^{-3}) - \ln x$$