Quiz #2 (CSE 400.001)

Thursday, March 25, 2004

Name: ____________________ E-mail: ____________________

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1. (10 points) Solve the following equation:

$$y''' + y' = \tan x$$

$\lambda^3 + \lambda = \lambda(\lambda^2+1) = 0 \Rightarrow \lambda = 0$ or $\lambda = \pm i$

$y_1 = 1,\ y_2 = \cos x,\ y_3 = \sin x$

(+2)

$$W = \begin{vmatrix} 1 & \cos x & \sin x \\ 0 & -\sin x & \cos x \\ 0 & -\cos x & -\sin x \end{vmatrix} = 1$$

$$W_1 = \begin{vmatrix} 0 & \cos x & \sin x \\ 0 & -\sin x & \cos x \\ 1 & -\cos x & -\sin x \end{vmatrix} = 1, \quad W_2 = -\cos x, \quad W_3 = -\sin x$$

(+3)

$$\begin{aligned} y_p &= y_1 \int \tan x\,dx + y_2 \int (-\cos x)\tan x\,dx + y_3 \int (-\sin x)\tan x\,dx \\ &= -y_1 \ln|\cos x| + y_2 \int (-\sin x)\,dx + y_3 \int \frac{-\sin^2 x}{\cos x}\,dx \\ &= -y_1 \cdot \ln|\cos x| + y_2 \cos x + y_3 \int (\cos x - \sec x)\,dx \\ &= -\ln|\cos x| + \underbrace{\cos^2 x + \sin^2 x}_{=1} - \sin x \cdot \ln|\sec x + \tan x| \end{aligned}$$

(+4)

$$\begin{aligned} y &= y_h + y_p \\ &= c_1 + c_2 \cos x + c_3 \sin x - \ln|\cos x| - \sin x \cdot \ln|\sec x + \tan x| \end{aligned}$$

(+1)

2. (15 points) Solve the following initial value problem:

$$y'' + 4y' + 4y = (3+x)e^{-2x}, \quad y(0) = 2, \; y'(0) = 5.$$

$\lambda^2 + 4\lambda + 4 = (\lambda+2)^2 = 0$

$y_h = c_1 e^{-2x} + c_2 x e^{-2x}$ (+1)

$y_p = (Ax^3 + Bx^2)e^{-2x}$ (+5)

$y_p' = [-2Ax^3 + (3A-2B)x^2 + 2Bx]e^{-2x}$ (+1)

$y_p'' = [4Ax^3 + (-12A+4B)x^2 + (6A-8B)x + 2B]e^{-2x}$ (+1)

$y_p'' + 4y_p' + 4y_p = (6Ax + 2B)e^{-2x} = (3+x)e^{-2x}$

$\therefore A = \frac{1}{6}, \; B = \frac{3}{2}$ (+3)

$y = c_1 e^{-2x} + c_2 x e^{-2x} + \left(\frac{1}{6}x^3 + \frac{3}{2}x^2\right)e^{-2x}$

$y' = -2c_1 e^{-2x} + c_2 e^{-2x} + x[\cdots]$ (+3)

$y(0) = c_1 = 2$

$y'(0) = -2c_1 + c_2 = 5$ $\Rightarrow$ $c_1 = 2$, $c_2 = 9$

$\therefore y = 2e^{-2x} + 9xe^{-2x} + \left(\frac{1}{6}x^3 + \frac{3}{2}x^2\right)e^{-2x}$ (+1)