Quiz #1 (CSE 400.001)

Wednesday, September 15, 2010

1. (6 points) Solve the following equation

$$(2\cos y + x^2)dx = x\sin y dy$$

$P = 2\cos y + x^2$, $Q = -x\sin y$

$\frac{1}{Q}(P_y - Q_x) = \frac{1}{-x\sin y}(-2\sin y + \sin y) = \frac{1}{x}$ (+3)

$F(x) = \exp\left(\int \frac{1}{x}dx\right) = \exp(\ln x) = x$

$(2x\cos y + x^3)dx - x^2\sin y\, dy = 0$

$U(x,y) = \int(-x^2\sin y)dy = x^2\cos y + l(x)$

$\frac{\partial U}{\partial x} = 2x\cos y + l'(x) = 2x\cos y + x^3$

$\therefore l(x) = \frac{1}{4}x^4 + c^*$

$\therefore U(x,y) = x^2\cos y + \frac{1}{4}x^4 = c$ (+3)

2. (4 points) Find the orthogonal trajectory of the following family of curves, where c is arbitrary.

$$y = \sqrt{x + c}$$

$y' = \frac{1}{2\sqrt{x+c}} = \frac{1}{2y}$ (+1)

For orthogonal trajectories,

$\frac{d\tilde{y}}{dx} = -2\tilde{y}$ (+1)

$\frac{d\tilde{y}}{\tilde{y}} = -2dx$

$\ln|\tilde{y}| = -2x + \tilde{c}$ (+1)

$\Rightarrow \tilde{y} = c^* \cdot e^{-2x}$ (+1)