

### Quiz #3 (CSE 400.001)

Monday, October 11, 2010

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1. (10 points) Solve the following initial value problem:

$$x^3 y''' + 7x^2 y'' - 2xy' - 10y = 0, \quad y(1) = 1, \quad y'(1) = -7, \quad y''(1) = 44.$$

$$m(m-1)(m-2) + 7m(m-1) - 2m - 10 = 0 \quad (+2)$$
$$(m+1)(m+5)(m-2) = 0$$

$$y_1 = x^2, \quad y_2 = \frac{1}{x}, \quad y_3 = \frac{1}{x^5} \quad (+2)$$

$$y = c_1 x^2 + c_2 \cdot \frac{1}{x} + c_3 \cdot \frac{1}{x^5} \quad (+1)$$

$$\left. \begin{aligned} y' &= 2c_1 x - c_2 \cdot \frac{1}{x^2} - 5c_3 \cdot \frac{1}{x^6} \\ y'' &= 2c_1 + 2c_2 \cdot \frac{1}{x^3} + 30c_3 \cdot \frac{1}{x^7} \end{aligned} \right\} (+2)$$

$$\begin{cases} c_1 + c_2 + c_3 = 1 \\ 2c_1 - c_2 - 5c_3 = -7 \\ 2c_1 + 2c_2 + 30c_3 = 44 \end{cases} \Rightarrow \begin{cases} c_1 = 0 \\ c_2 = -\frac{1}{2} \\ c_3 = \frac{3}{2} \end{cases} \quad (+2)$$

$$\therefore \underline{y = -\frac{1}{2x} + \frac{3}{2x^5}} \quad (+1)$$

2. (10 points) Solve the following system of ODEs:

$$\begin{aligned}y_1' &= y_1 + 2y_2 + t^2 \\ y_2' &= 2y_1 + y_2 - t^2\end{aligned}$$

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, \det(A - \lambda I) = (\lambda + 1)(\lambda - 3) = 0 \quad (+1)$$

$$\lambda_1 = -1, \mathbf{x}^{(1)} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}; \quad \lambda_2 = 3, \mathbf{x}^{(2)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad (+1)$$

$$\mathbf{y}_h = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{3t} \quad (+1)$$

$$\mathbf{y} u' = \begin{bmatrix} e^{-t} & e^{3t} \\ -e^{-t} & e^{3t} \end{bmatrix} \begin{bmatrix} u_1' \\ u_2' \end{bmatrix} = \begin{bmatrix} t^2 \\ -t^2 \end{bmatrix} \quad (+2)$$

$$\begin{bmatrix} u_1' \\ u_2' \end{bmatrix} = \frac{1}{2e^{2t}} \begin{bmatrix} e^{3t} & -e^{3t} \\ e^{-t} & e^{-t} \end{bmatrix} \begin{bmatrix} t^2 \\ -t^2 \end{bmatrix} = \begin{bmatrix} t^2 e^t \\ 0 \end{bmatrix} \quad (+1)$$

$$u_1 = \int_0^t \tilde{t}^2 e^{\tilde{t}} d\tilde{t} = t^2 e^t - 2te^t + 2e^t - 2 \quad (+2)$$

$$u_2 = 0 \quad (+1)$$

$$\begin{aligned}\mathbf{y}_p &= (t^2 e^t - 2te^t + 2e^t - 2) \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} (t^2 - 2t + 2) \\ &= \begin{bmatrix} 1 \\ -1 \end{bmatrix} (t^2 - 2t + 2 - 2e^{-t})\end{aligned}$$

$$\therefore \mathbf{y} = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{3t} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} (t^2 - 2t + 2)$$

(+1)