Quiz #4 (CSE 400.001)

Monday, November 8, 2010

Name: ______________________ **E-mail:** ______________________

Dept: ______________________ **ID No:** ______________________

1. (10 points) Find the inverse Laplace transform of the following function

$$F(s) = \frac{1}{(s+1)(s^2+1)}$$

$$f(t) = e^{-t} * \sin t$$
$$= \sin t * e^{-t} \quad +2$$
$$= \int_0^t \sin \tau \cdot e^{-(t-\tau)} \, d\tau \quad +2$$
$$= e^{-t} \int_0^t e^{\tau} \cdot \sin \tau \, d\tau \quad +2$$
$$= e^{-t} \left[\frac{1}{2} e^{\tau} (\sin \tau - \cos \tau) \Big|_{\tau=0}^{t} \right] \quad +2$$
$$= \frac{1}{2}(\sin t - \cos t) + \frac{1}{2} e^{-t}$$
+2

2. (5 points) Find the Laplace transform of the following function

$$f(t) = \int_0^t \sin(t-\tau)\cos\tau \, d\tau$$

$f(t) = \sin t * \cos t$ +2

$F(s) = \frac{1}{s^2+1} \cdot \frac{s}{s^2+1}$

$= \frac{s}{(s^2+1)^2}$ +3

3. (10 points) Find the inverse Laplace transform of the following function

$$\ln \frac{(s+a)^2}{(s+b)^3}$$

$F(s) = 2\ln(s+a) - 3\ln(s+b)$

$F'(s) = \frac{2}{s+a} - \frac{3}{s+b}$ +5

$-tf(t) = 2e^{-at} - 3e^{-bt}$

$f(t) = \frac{1}{t}\left[3e^{-bt} - 2e^{-at}\right]$ +5

4. (10 points) Solve the following ODE using the power series method

$$y' = y + x$$

$$y = \sum_{m=0}^{\infty} a_m x^m$$

$$y' = \sum_{m=1}^{\infty} m a_m x^{m-1}$$

(+2)

$$a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \cdots$$

$$= a_0 + (a_1+1)x + a_2 x^2 + a_3 x^3 + \cdots$$

(+2)

$$a_1 = a_0, \quad a_2 = \frac{1}{2}(a_1+1) = \frac{1}{2}(a_0+1)$$

$$a_3 = \frac{1}{3}a_2 = \frac{1}{3!}(a_0+1)$$

$$a_4 = \frac{1}{4}a_3 = \frac{1}{4!}(a_0+1)$$

(+3)

$$\therefore y = a_0 + a_0 x + (a_0+1)\left[\frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots\right]$$

$$= a_0 + a_0 x + (a_0+1)\left[e^x - 1 - x\right]$$

$$= a_0 e^x + e^x - 1 - x$$

(+3)

$$= (a_0+1)e^x - 1 - x$$