# Quiz #1 (CSE 400.001)

Monday, September 19, 2011

**Name:** ____________________ **E-mail:** ____________________

**Dept:** ____________________ **ID No:** ____________________

1. (6 points) Solve the following equation:

$$(x+y)dx + dy = 0.$$

$$P(x,y) = x+y, \quad Q(x,y) = 1$$

$$R(x) = \frac{1}{Q}(P_y - Q_x) = 1$$

(+1)

$$F(x) = \exp\left(\int 1\, dx\right) = e^x$$

$$e^x(x+y)dx + e^x dy = 0$$

(+2)

$$u(x,y) = \underline{e^x \cdot y + l(x)}$$ (+1)

$$u_x = e^x \cdot y + l'(x) = e^x \cdot (x+y)$$

$$\therefore l(x) = \int x e^x dx = (x-1)e^x + C_1$$

(+1)

$$\therefore u(x,y) = (x+y-1)e^x = C$$ (+1)

2. (5 points) Solve the following equation:

$$(y\cos x + 2xe^y)dx + (\sin x + x^2e^y - 1)dy = 0$$

$M = y\cos x + 2xe^y$, $N = \sin x + x^2e^y - 1$ (+1)

$\frac{\partial M}{\partial y} = \cos x + 2xe^y = \frac{\partial N}{\partial x}$ : exact! (+1)

$u = \int M dx + k(y) = y\sin x + x^2e^y + k(y)$ (+1)

$\frac{\partial u}{\partial y} = \sin x + x^2e^y + k'(y) = N = \sin x + x^2e^y - 1$ (+1)

$\therefore k(y) = -y + c^*$

$\therefore u(x,y) = y\sin x + x^2e^y - y = c$ (+1)

3. (4 points) Solve the following initial value problem:

$$(2\cos 2x)dx - (2y+3)dy = 0, \ y(0) = 1.$$

$(2\cos 2x)dx = (2y+3)dy$ (+1)

$\sin 2x = y^2 + 3y + c$ (+1)

$0 = 1 + 3 + c$

$\therefore c = -4$ (+1)

$\therefore \sin 2x - (y^2+3y) + 4 = 0$ (+1)

4. (5 points) Solve the following initial value problem:

$$x^2y'' + xy' - y = 0, \quad y(1) = 1,\ y'(1) = 7.$$

$$m(m-1)+m-1=0$$
$$m^2=1 \Rightarrow m=\pm 1$$
(+2)

$$y = c_1 x + c_2 \cdot \frac{1}{x}$$
$$y' = c_1 - c_2 \cdot \frac{1}{x^2}$$
(+1) $\Rightarrow$

$$c_1+c_2=1$$
$$c_1-c_2=7$$
$$\therefore c_1=4,\ c_2=-3$$
(+1)

$$\therefore y = 4x - 3 \cdot \frac{1}{x}$$
(+1)

5. (5 points) Solve the following initial value problem:

$$y'' + 1.4y' + 0.49y = 0, \quad y(0) = 1,\ y'(0) = 2.$$

$$\lambda^2 + 1.4\lambda + 0.49 = 0$$
$$(\lambda+0.7)^2 = 0$$
(+1)

$$\therefore y = c_1 e^{-0.7x} + c_2 x e^{-0.7x} \Rightarrow c_1 = 1$$
(+1)

$$y' = -0.7e^{-0.7x} + c_2 e^{-0.7x} - 0.7c_2 x e^{-0.7x}$$
(+1)

$$-0.7 + c_2 = 2 \Rightarrow c_2 = 2.7$$
(+1)

$$\therefore y = e^{-0.7x} + 2.7xe^{-0.7x}$$
(+1)

6. (5 points) Solve the following equation:

$$y'' + 4y' + 7y = 0.$$

$\lambda^2 + 4\lambda + 7 = 0$ (+1)

$(\lambda+2)^2 + 3 = 0$ (+1)

$\lambda = -2 \pm \sqrt{3}\, i$ (+1)

$\therefore y = e^{-2x}\left(A\cos\sqrt{3}x + B\sin\sqrt{3}x\right)$

(+2)